Intergenerational Resale Premium under Experience Effects [*], [†]

Bogdan Klishchuk [‡]

October 21, 2022

A documented experience effect—imperviousness to information that is not experience-based—generates belief heterogeneity that in the absence of short-selling gives rise to an intergenerational resale premium in the asset price. Apparently innocuous differences in histories which investors live through generate significant resale premiums. To this end, we develop a stripped-down overlapping-generations model with uncertainty only about the terminal time, the only time the asset pays a dividend. Most investors' histories end before the asset pays out and differ only in the birth date. Imperviousness to information that is not experience-based is there solely in that posteriors come from Bayesian updating from birth only. Yet a resale premium appears, while investors retrade before reaching their terminal age.

Keywords: resale premium; speculative premium; bubble; buy-and-hold price; experience effect; overlapping generations

JEL Codes: D83; D84; D87; D91; G12; G41

[*]Financial support by Deutsche Forschungsgemeinschaft through CRC TRR 190 (project number 280092119) is gratefully acknowledged.

[†]The author extends sincere thanks for their valuable feedback to Ulrich Horst, Stefanie Schräder, Rabee Tourky, Jan Werner, as well as audiences at the Econometric Society European Meeting 2022 and the XXX European Workshop on Economic Theory, where the author presented this paper under the title "Buy-and-resell Overpricing and Investor Experience". Declarations of interest: none.

[‡]Humboldt University of Berlin, Department of Mathematics, Unter den Linden 6, 10099 Berlin, Germany; e-mail address: klishchb@hu-berlin.de.

1. Introduction

Investors' beliefs exhibit experience effects documented by a growing empirical literature, where one of the key findings is imperviousness to information that is not experience-based (Malmendier, 2021). We integrate this intergenerational experience effect into the resale-option theory of bubbles[1] relative to buy-and-hold prices under the joint effects of short-sales constraints and belief heterogeneity, modeled in a variety of ways since Harrison and Kreps (1978). Apart from an informal discussion in Xiong and Yu (2011), the experience effects are new to this literature, which has focused on three mechanisms of belief heterogeneity: overconfidence as in Scheinkman and Xiong (2003), heterogeneous priors with correct updating as in Morris (1996), and simply dogmatic Markov beliefs independent of past dividends as in Harrison and Kreps (1978). A more recent addition is coarse reasoning introduced by Steiner and Stewart (2015), but they study a different kind of distortion—prices being the same in any two states categorized together by investors—rather than resale premiums. Regardless of the mechanism, Werner (2020) says that a sufficient condition for a resale premium to appear is that no investor be from some time onwards at least as optimistic about holding the asset forever as everyone else. We identify an overlapping-generations analogue of this sufficient condition and show that the resale premium is very sensitive to the new mechanism—imperviousness of investors' beliefs to information that is not experience-based.

Apparently innocuous differences in histories which investors live through generate large resale premiums relative to buy-and-hold prices, meaning that investors retrade before reaching their terminal age. To keep differences in investors' histories to a minimum, we develop a stripped-down overlapping-generations model with uncertainty only

[1]The difference between the equilibrium and buy-and-hold prices originally went under the names of speculative premium (Harrison and Kreps, 1978; Morris, 1996) and resale premium (Morris, 1992), which we also adopt. The term buy-and-hold price (Nutz and Scheinkman, 2020) condenses "what any class would be willing to pay for the stock if obliged to hold it [to horizon]" (Harrison and Kreps, 1978; Allen and Gorton, 1993) and even "the most optimistic fundamental valuation" (Morris, 1996).

about the terminal time, the only time the asset pays a dividend. This way, most investors' histories end before the asset pays the dividend and differ only in the birth date. To keep room for belief heterogeneity to a minimum, the only experience effect is that investors' posterior beliefs come from updating some common beliefs from birth only, which is a form of imperviousness to information that is not experience-based. Coexisting investors' beliefs are still in general heterogeneous because the only nondegenerate distribution invariant under conditioning on lapsed time (lack-of-memory property in probability theory) is the exponential distribution (see, e.g., Billingsley, 1995).

Relative to previous resale-option models of bubbles built on Harrison and Kreps (1978), we focus on an intergenerational resale premium but retain their usual features: belief heterogeneity, short-sales constraints, and also risk neutrality. Belief heterogeneity is always critical for a resale premium to appear, the form of short-sales constraints is unimportant (Morris, 1996), while risk neutrality is convenient but not strictly necessary (Hong et al., 2006; Morris, 1996; Morris, 1992). Alongside these usual ones, our model has three distinguishing features discussed next.

The first is a particular, but represented in the real world, type of risky asset with random, from investors' perspective, maturity date. An example is a credit default swap that pays out when the underlying bond defaults (for an empirical inquiry into this market, see, e.g., Oehmke and Zawadowski, 2017). But our model is also relevant to shares and can represent the prevalence of non-dividend-paying stocks during the real-world episodes considered to be instances of speculative bubbles. For example, the dotcom bubble happened at the time of disappearing dividends (Michaely and Moin, 2021). In 2021, the GameStop frenzy occurred even though the company stopped paying dividends in 2019 and as of 2021 was projected to report losses for two years (Jarvis, 2021; Newburger, 2019). For modeling bubbly resale premiums, therefore, our assumption of delayed dividends, paid only at the terminal time as a liquidation value, is in general rather appealing but features only in a few papers in the previous literature (Nutz and Scheinkman, 2020; Allen et al., 1993).

The second feature is the overlapping-generations population of investors. It is natural but may cause confusion between intergenerational resale premiums and rational bubbles that for a long time, before Santos and Woodford (1997), were exclusive to overlapping generations. Those are different things. Rational bubbles are deviations from the fundamental theorem of asset pricing, whereas intergenerational resale premiums are relative to buy-and-hold prices. The former can occur either with or without overlapping generations (Santos and Woodford, 1997). The intergenerational aspects aside, resale premiums with finitely-lived investors also appear in the models of Nutz and Scheinkman (2020), Berestycki et al. (2019), Allen et al. (1993), and Allen and Gorton (1993).

The third, and new to this literature, is of course imperviousness of investors' beliefs to information that is not experienced-based. Outside this literature, however, this experience effect has a long research history from different perspectives. It falls under the realm of availability bias in psychology-inspired literature—one of the key systematic biases identified in the seminal paper by Tversky and Kahneman (1974). As Malmendier (2021) put it, from a psychology perspective "people tend to overweight events that come to mind easily, that is, are 'available' to them, and personal experiences are a catalyst for this availability". Furthermore, it has a neuroscience foundation, linked to experience effects in finance by Malmendier (2021), in that more frequently used neurons are more connected and disuse withers previously formed synapses in the brain. In our setting, the information flow is simple enough to model this experience effect as information neglect in that investors do not consider what happened before birth. This simplification has already proved insightful in modeling other aspects of asset pricing (Ehling et al., 2017; Nakov and Nuño, 2015), while investors at least partially consider what happened before birth in Malmendier et al. (2020), Collin-Dufresne et al. (2016), and Schraeder (2016).

Modeling an intergenerational resale premium under experience effects is therefore also appealing, especially as the resale-option theory transitioned from heterogeneous priors (Harrison and Kreps, 1978; Morris, 1996) to embracing biased beliefs since Scheinkman and Xiong (2003).

Our model has five primitives: the terminal time; the remaining time, as perceived at birth by every investor, in the form of a random variable; the terminal age; the trade-frequency parameter; and the interest rate. The rest is further interpretation. We have already mentioned the asset; it pays a one-shot dividend of \$1 at the terminal time. Before that, overlapping generations come and go: one investor is born each time point and stays until the terminal age or the terminal time, whichever is sooner. Time is continuous, backward infinite so that the age profile of coexisting investors is time-invariant, and possibly forward infinite depending on the terminal-time parameter. Trade, however, is discrete to avoid dealing with equilibria where the asset might change hands continuously, but our results on intergenerational resale premiums are asymptotic as the market tends to opening continuously. Investors are risk-neutral and decide whether to buy/sell the asset or not at an endogenous price every time the market opens for trade, but they cannot sell the asset short.

An attractive feature of our model is that we can require the equilibrium price to be time-invariant, because coexisting investors' age profile is time-invariant and their posteriors depend only on their age. In this context, summarizing the adapted equilibrium criterion to overlapping generations from Harrison and Kreps (1978), where, characteristically, only the most optimistic investors hold the asset, affords simple wording. An equilibrium price is one that makes the most optimistic investor(s) break even, but who the most optimistic are depends on the sought equilibrium price via expected resale proceeds corresponding to optimal holding durations. If, on the contrary, the most optimistic investors did not break even, that would mean either excessive expected discounted return for the most optimistic or expected loss for all. Equilibrium buyers/holders at a particular time the market opens for trade can be any subset of the most optimistic investors given the equilibrium price as a (provisional) resale price. Less optimistic investors simply do not, for whatever reason by assumption, engage in short-selling of the asset and do not create excess supply in this way. Investors do not have budget constraints, but at the equilibrium price we find all of the asset can cost at most $1, and thus we do not need the infinite-wealth assumption from the previous literature.

The benchmark buy-and-hold price is what an equilibrium price would be if buyers had to meet a life-long no-retrade constraint and only then could resell at this sought price if the dividend were still unpaid. In the previous literature with infinitely-lived investors and without overlapping generations, the benchmark collapses to the highest estimate of the expected discounted return of holding the asset forever among all investors. The few existing finite-horizon models, without overlapping generations, replace holding forever in the definition of benchmark price simply with holding to the terminal time (Nutz and Scheinkman, 2020; Berestycki et al., 2019; Allen et al., 1993; Allen and Gorton, 1993). For overlapping generations that can live forever, which we make a special case given that in equilibrium only the most optimistic hold the asset, holding forever still makes sense. But the finite-lifetime case of our overlapping-generations model creates a difficulty in defining benchmark for studying intergenerational resale premiums in this way, because holding the asset forever becomes problematic. Our approach overcomes this difficulty.

Our results on intergenerational resale premiums are asymptotic in the sense that we take these discrete-trade equilibrium and buy-and-hold prices to their continuous-trade limits before comparing them. Mild assumptions on the investors' common beliefs at birth, which are about how long is left until the terminal time, are in place for the unique discrete-trade equilibrium and buy-and-hold prices to exist and to converge. To give our results on intergenerational resale premiums in this continuous-trade limit, we need the investors' common beliefs at birth about the waiting time to have a density.

Belief heterogeneity is necessary but not sufficient for a resale premium to appear, although most models parameterize beliefs in such a way that without short-selling their heterogeneity is also sufficient; exceptions are Morris (1996) and Werner (2020). An intergenerational resale premium appears if and only if the most senior investors are not instantaneously the most optimistic, with instantaneous optimism being measured by the hazard rate of the investors' common beliefs at birth. The equilibrium price can even hit the upper bound of \$1 on any reasonable fundamental value as if the one-shot dividend of \$1 were imminent and there were no uncertainty about its timing. The price is as high as this in the continuous-trade limit when newborn investors' beliefs are instantaneously sufficiently optimistic in the sense that the hazard rate, viewed as a function of age, diverges to infinity at zero. At the same time, the buy-and-hold price can be arbitrarily low, implying frequent retrading at an arbitrarily large resale premium in the equilibrium price. A simple example is to take for the investors' common beliefs at birth a gamma distribution with suitable parameters.

Our result that the equilibrium price can be as high as any reasonable fundamental value is new to the resale-option theory initiated by Harrison and Kreps (1978). Previous results on equilibrium prices as high as this come from models of (non-dividend-paying) money (Tirole, 1985; Samuelson, 1958) or asymmetrically informed greater fools (Allen et al., 1993; Barlevy, 2015; Liu and Conlon, 2018). One interpretation of our results is that retrading (before horizon) and overpricing can come from everyone being equally fool but of different age as opposed to what happens in the greater-fool models.

The paper is organized as follows. Section 2 presents the model, including the equilibrium concept defining equilibrium price. Buy-and-hold price is a benchmark defined in Section 3. To compare them, Section 4 gives a necessary and sufficient condition for a nonzero resale premium in the limit as the market tends to opening continuously, along the way addressing their existence, uniqueness, and convergence. In Section 5, the equilibrium price's continuous-trade limit hits the bound of what any reasonable fundamental value could possibly be, while the buy-and-hold price is not just lower, but arbitrarily so, in the limit, which are our final and main results. The proof of all but one proposition comes at the end as Appendix A.

2. Model

The model is stripped-down overlapping generations without short-selling and with an adapted equilibrium concept to overlapping generations from the resale-option models of bubbles built on Harrison and Kreps (1978). A distinguishing feature of this equilibrium concept is that only the most optimistic investors, given how they forecast dividends and equilibrium (resale) prices, hold the asset. We make time continuous, backward infinite so that the age profile of coexisting investors is time-invariant, and possibly forward infinite depending on the terminal-time parameter. Risk-neutral investors decide whether to buy/sell the asset or not at an endogenous steady-state price under uncertainty about the terminal time, the only time the asset pays a dividend. After first going over the model's primitives (Section 2.1), we take on the equilibrium concept (Section 2.2) and then give the outlook (Section 2.3) of smoothness and regularity assumptions made later on.

2.1. Primitives and Basics

In total, our model has five primitives:

(1) the terminal time $\tau \in (-\infty, \infty]$ at which an asset pays a one-time dividend of \$1;

(2) the remaining time, as perceived at birth by every investor, in the form of a random

variable[2] W taking values in $(0, \infty]$;

(3) the terminal age $T \in (0, \infty]$;

(4) the trade-frequency parameter $\Delta \in (0, T)$;

(5) the interest rate $r \in (0, \infty)$.

An asset pays a one-shot dividend of \$1 at the possibly infinite terminal time τ, where we think of the dividend at infinity as no dividend. Investors do not know (and are uncertain about) the terminal time τ until it comes, but this true terminal time τ is not random. Before it comes, one investor is born each time point and stays until the terminal age T or the terminal time τ, whichever is sooner: at each $t \in (-\infty, \tau)$, a new one is born, and unless $T = \infty$ the time is up for the one born at time $t - T$. Investors start out with the same beliefs at birth, namely that the time left is the random variable W.

Investors' beliefs exhibit imperviousness to information that is not experience-based in that W is a primitive and does not come from updating some priors, though one can put it in the standard language of heterogeneous priors over a larger space. As Morris (1995) argues more generally, "it would be possible to interpret such systematic biases as a consequence of different prior beliefs on some larger state spaces. Such an interpretation would be misleading." But key experience effects, such as imperviousness to information that is not experienced-based, are not just systematic biases; they have a neuroscience foundation (Malmendier, 2021). That is why investors' beliefs at birth represented by W are a primitive and do not come from updating some prior beliefs, but they update after birth in the standard Bayesian way at every nonterminal time. In other words, at each time $s \in (-\infty, \tau)$, posterior expectations (if well-defined) of the investor born at an arbitrary time $t \in (-\infty, s)$ of functions of the perceived remaining

[2]We consider extended-valued random variables defined on a probability space $(\Omega, \mathcal{F}, \mathrm{P})$. For a formal definition with extended values, see Chung (2001). We use the convention $e^{-\infty} = 0$.

time W are expectations conditional on the event that $W > s - t$. The information flow is so simple that the posterior depends only on the investor's age $s - t$ via conditioning on survival up to this age, assumed to have a nonzero perceived probability at birth:

Assumption 1. Nonzero perceived probability at birth of survival up to the terminal age T in the sense that all $x \in [0, T] \backslash \{\infty\}$ satisfy $\mathrm{P}(W > x) \neq 0$.

Investors can trade every Δ time units at the time points

$$\{0, \Delta, -\Delta, 2\Delta, -2\Delta, 3\Delta, -3\Delta, \dots\} \cap (-\infty, \tau),$$

which are when the market opens for trade. Finally, investors can borrow and lend at the constant rate r, which will serve as a discount rate.

2.2. Equilibrium

The same story applies to every time the market opens for trade, because apart from the uncertain terminal time our model is so stationary that we parameterize coexisting investors by their age $x \in [0, T] \backslash \{\infty\}$. Now since the investors' posteriors depend only on their age, we require the sought equilibrium price $p \in [0, 1]$ to be time-invariant. In turn, we also find equilibrium asset holders in terms of their age and intergenerational transactions that can keep the asset in their hands. Our adapted equilibrium concept to this context from Harrison and Kreps (1978) retains its characteristic feature, which is that only the most optimistic investors hold the asset. In our context, the equilibrium criterion boils down to essentially just an equation (1) in the price p, from which we can directly find who can hold the asset and when as follows (existence conditions are later on in Section 4.1).

In the steady-state equilibrium condition (1), the max picks up investors in terms of their age x at which their posteriors are such that they perceive the best holding duration

y ever, subject to lifetime and trade-frequency constraints. They ensure that the holding duration y falls within the remaining lifetime $T-x$ in the sense that $y \leq T-x$ and agrees with how often the market opens for trade in the sense that $y \in \{\Delta, 2\Delta, 3\Delta, \ldots, \infty\}$. Investors can participate in the market only when the remaining lifetime $T-x$ from reaching the age of x is long enough for the market to reopen for trade at least once in the sense that $T-x \geq \Delta$, hence overall the max is over

$$x \in [0, T-\Delta] \backslash \{\infty\} \quad \text{and } y \in \{\Delta, 2\Delta, 3\Delta, \ldots, \infty\} \cap [\Delta, T-x] \, .$$

The expected payoff (to risk-neutral investors) at age x from holding duration y is the (posterior) expected discounted return of either getting the dividend within y time units or else selling the asset in y time units. The max in the equilibrium condition (1) is over these payoffs and in case the price p^*_Δ is the sought equilibrium price picks up equilibrium asset holders in terms of their age x and their equilibrium trading strategies y. In this sense, investors can in equilibrium hold the asset when they are the most optimistic in life given the equilibrium price as a resale price (conditional on the dividend being still unpaid). Trade at a particular time the market opens can in equilibrium be anything that passes the asset to investors reaching any such age. The equilibrium condition (1) simply requires the price to equal the maximum payoff, which itself depends on this price via expected resale proceeds, over investors in terms of their age and trading strategies. If the equilibrium price differed from the expected discounted return of the best trading strategy across all investors when the (provisional) resale price is this equilibrium price, there would be either shortage or excess supply. Implicitly, we have also made the following assumptions:

(i) short-sales constraints (investors simply do not, for whatever reason, engage in short-selling of the asset and do not drive the price down in this way);

(ii) investors can afford to buy all of the asset (which can cost at most \$1, though);

(iii) within a particular nonterminal time, first everyone learns that the asset does not pay, then a new investor arrives, and then the market either opens for trade or not.

The interpretation aside, we simply have:

Definition 1 (Equilibrium Price)**.** We say that a price $p^*_\Delta \in [0,1]$ is an equilibrium price if it equals the expected discounted return from the best holding duration y within lifetime and trade-frequency constraints across all investors in terms of their age x in the sense that

$$p^*_\Delta = \max_{\substack{x\in[0,T-\Delta]\setminus\{\infty\} \\ y\in\{\Delta,2\Delta,3\Delta,\ldots,\infty\}\cap[\Delta,T-x]}} \mathrm{E}\left(e^{-r(W-x)}I_{\{W\le x+y\}} + p^*_\Delta e^{-ry}I_{\{W>x+y\}}\middle|W>x\right) \tag{1}$$

and the maximum exists.

2.3. Outlook of Smoothness and Regularity Assumptions

We ensure the existence of an equilibrium price and other unknowns by several unsurprising smoothness and regularity assumptions, which we place so that only what comes after an assumption may need that assumption. They require the perceived remaining time W to have a density (Assumption 4) that is continuous (Assumption 5) and has enough of a tail (Assumption 1) to, in particular, induce on $(0,T)$ a hazard rate assumed to converge at the endpoints (Assumption 6). This is all we assume for the finite-lifetime case $T<\infty$, because then Assumption 3 is redundant and from the combined list we can omit continuity of the distribution (Assumption 2) strengthened later with the density (Assumption 4). With the finite-lifetime resale-option model at hand, an extension to infinitely-lived investors is of less importance and just helps, in our view, see that things work analogously in the finite-lifetime case (in particular, note Proposition 1 in Section 3). For this reason, we are not concerned about the strong assumption of eventual first-order stochastic dominance (Assumption 3) made to ensure the existence of an equilibrium price when $T = \infty$, which involves noncompact maximization.

3. Buy-and-hold Price versus Equilibrium Price

Buy-and-hold price is the benchmark used in the resale-option theory of bubbles built on Harrison and Kreps (1978), but we need an extension to overlapping generations. The difficulty in the finite-lifetime case is to consider resale values even to define buy-and-hold price, which we can no longer take to be the highest estimate of the expected discounted return of holding the asset forever among investors. Instead, we define it as what an equilibrium price would be if buyers of the asset had to meet a no-retrade constraint and only then could resell at this price if the dividend were still unpaid (Definitions 2–3 followed by verbal explanations):

Definition 2 (No-retrade Constraint for Benchmark Equilibrium)**.** The correspondence from the participating investors' age interval

$$Y_\Delta : [0, T - \Delta] \backslash \{\infty\} \twoheadrightarrow \{\Delta, 2\Delta, 3\Delta, \ldots, \infty\}$$

defined by (longer no-retrade durations for younger investors)

$$Y_\Delta(x) = \begin{cases} \{\Delta\} & \text{if } T - 2\Delta < x \leq T - \Delta, \\ \{\Delta, 2\Delta\} & \text{if } T - 2\Delta = x, \\ \{2\Delta\} & \text{if } T - 3\Delta < x < T - 2\Delta, \\ \{2\Delta, 3\Delta\} & \text{if } T - 3\Delta = x, \\ \{3\Delta\} & \text{if } T - 4\Delta < x < T - 3\Delta, \\ & \vdots \\ \{\infty\} & \text{if } T = \infty \end{cases}$$

is a no-retrade constraint.

In this constraint for the definition of benchmark buy-and-hold price, either infinite lifetime $T = \infty$ is the case and the no-retrade duration is forever or else the no-retrade duration depends on remaining lifetime as follows: one trading round at the top age bracket, one or two at the threshold for the second top bracket (for some continuity of the correspondence), two at the second top bracket itself, and so on to as many trading rounds as possible within the finite lifetime $T < \infty$ at the lowest age bracket. Essentially, to buy and hold means to hold the asset for the rest of the buyer's investment horizon and then resell at the sought buy-and-hold price as long as the dividend is still unpaid:

Definition 3 (Buy-and-hold Price)**.** We say that a price $\bar{p}_\Delta \in [0, 1]$ is a buy-and-hold price if it equals the most optimistic expected discounted return from restricted trading under the no-retrade constraint (Definition 2) across all investors in terms of their age x in the sense that

$$\bar{p}_\Delta = \max_{\substack{x \in [0, T-\Delta] \setminus \{\infty\} \\ y \in Y_\Delta(x)}} \mathrm{E}\left(e^{-r(W-x)} I_{\{W \leq x+y\}} + \bar{p}_\Delta e^{-ry} I_{\{W > x+y\}} \middle| W > x\right) \tag{2}$$

and the maximum exists.

A bridge to the previous literature is the infinite-lifetime case $T = \infty$, when the buy-and-hold price is simply the most optimistic expected discounted return of holding the asset forever among all investors. Indeed, in the case $T = \infty$, when the participating investors' age interval always constitutes $[0, \infty)$, condition (2) becomes

$$\bar{p}_\Delta = \max_{x \in [0,\infty)} \mathrm{E}\left(e^{-r(W-x)} \middle| W > x\right) \tag{3}$$

(a buy-and-hold price exists if and only if the maximum exists, as addressed more generally in Section 4.1). The reason the previous literature defines benchmark price as the highest expected discounted value of future dividends among investors is that it reflects the willingness to pay for the asset if obliged to hold it forever, without reselling. Viewing this buy-and-hold price as this willingness to pay is key to the resale-option bubble theory: if the price exceeds everyone's expected discounted value of dividends, whoever buys it will resell despite being infinitely-lived. Indeed, in the case $T = \tau = \infty$ the buyer does not only plan to stop waiting for the dividend at some future time point, but actually stops and resells. Proposition 1 below covers this and the general case to show that things work analogously in the finite-lifetime case $T < \infty$ under our choice of the no-retrade constraint and resale value for the definition of buy-and-hold price (Definitions 2–3). If the buyer held the asset for the rest of the investment horizon, that would include holding it for the final trading round in life, which would trivially satisfy the no-retrade constraint and mean that the price is a buy-and-hold price. The argument for the more familiar infinite-lifetime case $T = \infty$ is longer and involves recursion:

Proposition 1 (Reselling Early within Investment Horizon)**.** *An equilibrium price p^*_Δ coincides with a buy-and-hold price if in this equilibrium it is optimal to meet the no-retrade constraint starting from some age $x \in [0, T - \Delta] \backslash \{\infty\}$ on, i.e., if for every $w \in$*

$\{0, \Delta, 2\Delta, 3\Delta, \dots\} \cap [0, T - \Delta - x]$ *there is a* $y \in \{\Delta, 2\Delta, 3\Delta, \dots, \infty\} \cap [\Delta, T - x - w]$ *such that* $(x + w, y)$ *is in the argmax in* (1).

Proof. $(T < \infty)$ At the age of x, the number of trading rounds within remaining lifetime is $\max Y_\Delta(x)$, the final trading round in the investor's life would be in $w = \max Y_\Delta(x) - \Delta$ time units, and by the hypothesis $(x + w, \Delta)$ is in the argmax in (1). This means that the equilibrium price p^*_Δ equals the expected discounted return of the investor of age $x + w$ of holding the asset for the final trading round. But reselling at the end trivially meets the no-retrade constraint in the sense that $\Delta \in Y_\Delta(x + w)$ and, thus, does not bind in condition (2) when $\bar{p}_\Delta = p^*_\Delta$, which is indeed a buy-and-hold price then.

$(T = \infty)$ The hypothesis implies the existence of a recursively defined sequence $\{y_n\}$ in $\{\Delta, 2\Delta, 3\Delta, \dots\}$ such that

$$\begin{aligned}
p^*_\Delta =& \mathrm{E}\left(e^{-r\left(W - x - \sum_{i=1}^{n-1} y_i\right)} I_{\left\{W \le x + \sum_{i=1}^{n} y_i\right\}} + p^*_\Delta e^{-r y_n} I_{\left\{W > x + \sum_{i=1}^{n} y_i\right\}} \middle| W > x + \sum_{i=1}^{n-1} y_i \right) \\
=& \mathrm{E}\left(e^{-r\left(W - x - \sum_{i=1}^{n-1} y_i\right)} I_{\left\{W \le x + \sum_{i=1}^{n} y_i\right\}} \middle| W > x + \sum_{i=1}^{n-1} y_i \right) \\
&+ p^*_\Delta e^{-r y_n} \frac{\mathrm{P}\left(W > x + \sum_{i=1}^{n} y_i\right)}{\mathrm{P}\left(W > x + \sum_{i=1}^{n-1} y_i\right)}.
\end{aligned}$$

By induction, we have

$$\begin{aligned}
p^*_\Delta =& \sum_{i=1}^{n} \mathrm{E}\left(e^{-r\left(W - x - \sum_{j=1}^{i-1} y_j\right)} I_{\left\{W \le x + \sum_{j=1}^{i} y_j\right\}} \middle| W > x + \sum_{j=1}^{i-1} y_j \right) \frac{\mathrm{P}\left(W > x + \sum_{j=1}^{i-1} y_j\right)}{e^{r \sum_{j=1}^{i-1} y_j} \mathrm{P}(W > x)} \\
&+ p^*_\Delta e^{-r \sum_{i=1}^{n} y_i} \frac{\mathrm{P}\left(W > x + \sum_{i=1}^{n} y_i\right)}{\mathrm{P}(W > x)}
\end{aligned}$$

for all indices n. By another induction, the first summation in this formula simplifies and turns it to

$$p^*_\Delta = \mathrm{E}\left(e^{-r(W - x)} I_{\left\{W \le x + \sum_{i=1}^{n} y_i\right\}} \middle| W > x \right) + p^*_\Delta e^{-r \sum_{i=1}^{n} y_i} \frac{\mathrm{P}\left(W > x + \sum_{i=1}^{n} y_i\right)}{\mathrm{P}(W > x)},$$

for all indices n. Passing to the limit shows that

$$p_{\Delta}^{*} = \mathrm{E}\left(e^{-r(W-x)}\middle|W > x\right),$$

hence the pair (x, ∞) is a maximizer in (1), and hence x is a maximizer in (3), confirming that p_{Δ}^{*} is a buy-and-hold price. □

4. Characterization of Pricing at Resale Premium

Belief heterogeneity is necessary but not sufficient for a (nonzero) resale premium to appear. We characterize pricing at a resale premium in terms of the hazard rate of the perceived remaining time W, although our main result concerns the sensitivity to how impervious these beliefs are to information that is not experience-based (Section 5). When viewed as a function of age, the hazard rate is a measure of instantaneous optimism about the dividend (conditional on it remaining unpaid). With this interpretation, our necessary and sufficient condition for a resale premium to appear is that the most senior investors are not instantaneously the most optimistic (Section 4.2 and an example in Section 4.3). Otherwise, the hazard rate can be, for instance, strictly increasing so that more senior investors are instantaneously more optimistic but a resale premium does not appear (an example in Section 4.3 as well). Under the benchmark exponential distribution, not only the hazard rate is constant so that our necessary and sufficient condition fails and a resale premium does not appear, but there is also no belief heterogeneity (Section 4.3). Our characterization of pricing at a resale premium is asymptotic in the sense that we take discrete-trade equilibrium and buy-and-hold prices, which exist and are unique, to their continuous-trade limits (Section 4.1) before comparing them.

4.1. Existence and Continuous-trade Limit

Mild assumptions on the perceived remaining time W are in place for unique discrete-trade equilibrium and buy-and-hold prices to exist and to converge. For the existence and uniqueness in the discrete-trade case, we need the following assumptions and terminology:

Definition 4 (Investors' Conditional Distributions—Posteriors—on Reaching Their Age)**.** The function $F(\cdot|\cdot) : \mathbb{R} \times [0, T) \to \mathbb{R}$ defined by

$$F(w|x) = \mathrm{P}(W - x \leq w | W > x)$$

is in the first variable the posterior distribution function (of an extended-valued random variable) given the second variable (age, x), and for every $x \in [0, T)$ we automatically have

$$\lim_{w\to\infty} F(w|x) = \mathrm{P}(W - x < \infty | W > x).$$

For $x = 0$, we denote $F(\cdot|0)$ by F and its limit at infinity $\lim_{w\to\infty} F(w)$ by $F(\infty)$.

Definition 5 (Extended First-order Stochastic Dominance)**.** For posteriors given survival up to any two $x, x' \in [0, T)$, we say that $F(\cdot|x)$ first-order stochastically dominates (FOSD) $F(\cdot|x')$ if for every (weakly) increasing function $u : (-\infty, \infty] \to \mathbb{R}$ we have

$$\begin{aligned}\int_{-\infty}^{\infty} u|_{\mathbb{R}}(w)\, dF(w|x) + u(\infty)\left(1 - \lim_{w\to\infty} F(w|x)\right) \\ \geq \int_{-\infty}^{\infty} u|_{\mathbb{R}}(w)\, dF(w|x') + u(\infty)\left(1 - \lim_{w\to\infty} F(w|x')\right).\end{aligned} \tag{4}$$

Assumption 2. Continuity of F on $[0, T] \setminus \{\infty\}$.

Assumption 3. Either $T < \infty$ or eventual first-order stochastic dominance in the sense that there exists an age threshold $\tilde{x} \in [0, T)$ such that $F(\cdot|x)$ first-order stochastically dominates $F(\cdot|\tilde{x})$ for all $x \in [\tilde{x}, T)$.

Here the important scenario is that of finitely-lived investors ($T < \infty$), while the infinite-lifetime case ($T = \infty$) is, from our perspective, for better compatibility with the resale-option models of bubbles built on Harrison and Kreps (1978). Only for this case's sake we involve first-order stochastic dominance (for extended-valued random variables), which we have defined analogously to that for finite-valued random variables (see, e.g., Mas-Colell et al., 1995). Just as first-order stochastic dominance of one money lottery over another means that expected utility of the first is at least as high as that of the second if one values more over less, so do we order this kind of beliefs. What Assumption 3 means in terms of the primitive objects is easy to state by replacing the phrase "$F(\cdot|x)$ first-order stochastically dominates $F(\cdot|\tilde{x})$" with inequality (4), the age x' with $\tilde{x}$, and the posteriors with their definitions in terms of F:

$$F(w|x) = \frac{\mathrm{P}(x < W \leq w + x)}{\mathrm{P}(W > x)} = \frac{F(w + x) - F(x)}{1 - F(x)}$$

if $w \in [0, \infty)$.

Example 1 (Stochastic Dominance under Incomplete Exponential Distribution)**.** Suppose that $0 < \mathrm{P}(W = \infty) < 1$ and conditionally on $W < \infty$ the distribution of W is exponential, i.e., there is a $\lambda \in (0, \infty)$ such that all $w \in [0, \infty)$ satisfy

$$\mathrm{P}(W \leq w | W < \infty) = 1 - e^{-\lambda w}$$

(an incomplete exponential distribution with parameters λ and $q = \mathrm{P}(W = \infty)$). The posterior given survival up to any age $x \in [0, T)$ remains incomplete exponential with the same exponential part but different (updated) probabilities of infinite time left till the terminal time, i.e.,

$$\mathrm{P}\left(W - x = \infty | W > x\right) = \frac{q}{q + (1-q)\, e^{-\lambda x}} \tag{5}$$

and all $w \in [0, \infty)$ satisfy

$$F\left(w|x\right) = \left(1 - \frac{q}{q + (1-q)\, e^{-\lambda x}}\right)\left(1 - e^{-\lambda w}\right).$$

Relative to the zero age at birth, at the age of x the posterior $F\left(\cdot|x\right)$ scales down everywhere according to this formula, because the posterior probability (5) of infinite remaining time goes up. In other words, the probability shifts to infinite remaining time from finite ones, hence, precisely as first-order stochastic dominance requires, expectations of increasing functions of the remaining time increase (Definition 5). This means that relative to the zero age at birth at the age of x the posterior $F\left(\cdot|x\right)$ is first-order stochastically dominant, and thus Assumption 3 holds simply with $\tilde{x} = 0$.

We return to this and other examples in Section 4.3, where we put three examples together in Table 1, Section 5 (Example 2), and Appendix B. Now we can prove the existence and uniqueness of (steady-state) equilibrium and buy-and-hold prices:

Proposition 2. *There exist:*

(i) a unique equilibrium price;

(ii) a unique buy-and-hold price.

The remaining question of this subsection is when these discrete-trade equilibrium and buy-and-hold prices have continuous-trade limits, in terms of which we characterize pricing at a resale premium in Section 4.2. For this we need another set of assumptions and terminology first.

Assumption 4. A density $f : \mathbb{R} \to [0, \infty)$ for F exists: f is integrable and all $w, w' \in \mathbb{R}$ with $w < w'$ satisfy

$$F(w') - F(w) = \int_{w}^{w'} f(z)\, dz.$$

Assumption 5. Continuity of f on $(0, T)$.

Assumption 4 allows us to introduce two important functions for stating our results on intergenerational resale premiums, proving them, and first obtaining the continuous-trade limits of the discrete-trade equilibrium and buy-and-hold prices (Proposition 3). These important functions are the hazard rate, as usual when uncertainty is about waiting time, and what we can call the hazard weight:

Definition 6 (Hazard Rate and Hazard Weight)**.** The hazard rate is the function $h : (0, T) \to [0, \infty)$, on the interior of the coexisting investors' age interval $[0, T] \backslash \{\infty\}$, defined by

$$h(x) = \frac{f(x)}{1 - F(x)}$$

(standard definition apart from the restricted domain). The hazard weight is the function $\tilde{h} : (0, T) \to [0, 1]$ defined by

$$\tilde{h}(x) = \frac{h(x)}{h(x) + r}.$$

Indeed, the name hazard weight we chose for this function stands for the relative weight of the hazard rate h and the interest rate r this function $\tilde{h}$ measures. Our necessary and sufficient condition for a resale premium to appear (Section 4.2) is on the hazard rate h, and so is our final Assumption 6 below, but they enter our proofs via the hazard weight $\tilde{h}$.

Assumption 6. One-sided limits of h at the endpoints of its domain or else divergence to infinity:

(i) h has a limit or tends to ∞ as $x \to 0^+$;

(ii) h has a limit or tends to ∞ as $x \to T^-$, where the convention is $\infty^- = \infty$.

Assumption 6 just disciplines the distribution F of the perceived remaining time W at the endpoints of the coexisting investors' age interval $[0, T] \setminus \{\infty\}$ in the sense of a sufficiently well-behaved hazard rate h (which we can express in terms of F itself and its derivative). It makes the analysis of the continuous-trade limits of the discrete-trade equilibrium and buy-and-hold prices, whose existence we can now prove, less tedious:

Proposition 3 (Continuous-trade Limits). *Consider (i) the equilibrium price p^*_Δ and (ii) buy-and-hold price $\bar{p}_\Delta$ found in Proposition 2 as (real) functions of the trade-frequency parameter Δ on $(0, T)$. They have (finite) right-hand limits at 0.*

An example in Appendix B shows that in this proposition the continuous-trade limit of the discrete-trade equilibrium price need not be a monotone limit. The relationship between resale premiums and trading frequency is complicated by the fact that the latter affects both the equilibrium and buy-and-hold prices via holding durations. We focus on resale premiums under frequent trade and compare the equilibrium and benchmark buy-and-hold prices in this continuous-trade limit in the following subsection.

4.2. Necessary and Sufficient Condition for Nonzero Premium

Our conditions do not only help relate intergenerational resale premiums to imperviousness of beliefs to information that is not experienced-based (Section 5), but also shed light on premium-neutral belief heterogeneity, complementing Morris (1996) and Werner (2020). In their models, nonzero resale premiums and belief heterogeneity are also not equivalent: sometimes only the latter occurs and does not give rise to a resale premium.

The former author characterizes pricing at a resale premium in a special case of Harrison and Kreps (1978) in terms of investors' prior beliefs about the probability $\theta \in [0,1]$ that the asset pays in any one period a \$1 dividend. Representing these prior beliefs by densities over the parameter θ, his necessary and sufficient condition for a resale premium to appear is that there be no investor with monotone-likelihood-ratio dominant prior beliefs. This condition remains sufficient for pricing at a resale premium under an arbitrary dividend process studied by Werner (2020). Although he considers other mechanisms of belief heterogeneity as well, he does not cover overlapping generations and imperviousness of investors' beliefs to information that is not experience-based, which we focus on. We obtain a different characterization of pricing at a resale premium, in terms of the hazard rate h of the perceived remaining time W. First we give two sufficient conditions for a resale premium to appear (Sections 4.2.1 and 4.2.2) in the continuous-trade limit of our discrete-trade model and then the necessity of the second one (Section 4.2.3).

4.2.1. 1st Sufficient Condition for Pricing at Resale Premium

To characterize pricing at a resale premium later in Sections 4.2.2–4.2.3, we first need to recover a standard sufficient condition that is robust to the mechanism of belief heterogeneity (unlike the likelihood-ratio condition). In the infinite-lifetime models of both Morris (1996) and Werner (2020), the general condition is that no investor be from some time onwards at least as optimistic about holding the asset forever as everyone else. The term used for this sufficient condition is, naturally, perpetual valuation switching, in which case, as Scheinkman and Xiong (2003) put it, investors' "fundamental beliefs cross". It works under an arbitrary dividend process (Werner, 2020) with overconfidence (Scheinkman and Xiong, 2003), heterogeneous priors with correct updating (Morris, 1996), and dogmatic Markov beliefs independent of past dividends (Harrison and Kreps, 1978). Building on this idea, we identify an analogue of this sufficient condition for overlapping generations and use it to prove the sufficiency of our main condition (Section 4.2.2), which is also necessary (Section 4.2.3). The former condition is in terms of personalized fundamental values, which in the benchmark infinite-lifetime case $T = \infty$ collapse to the expected discounted returns of holding the asset forever. We extend them to the expected discounted returns of holding for the rest of life and then actually reselling as long as the dividend is unpaid, but at the buy-and-hold price's continuous-trade limit as the resale price, which we take from Proposition 3 and denote by

$$\bar{p}_0 = \lim_{\Delta \to 0^+} \bar{p}_\Delta.$$

In our model, the resulting personalized fundamental values (conditional on the dividend remaining unpaid) are a function of age, which we call by the shorter name of a fundamental valuation:

Definition 7 (Fundamental Valuation)**.** The fundamental valuation is the function of age $V : [0, T] \setminus \{\infty\} \to \mathbb{R}$ defined by

$$\begin{aligned} V(x) &= \mathrm{E}\left(e^{-r(W-x)} I_{\{W \le T\}} + \bar{p}_0 e^{-r(T-x)} I_{\{W > T\}} \middle| W > x\right) \\ &= \frac{1}{1 - F(x)} \left(\int_x^T e^{-r(w-x)} dF(w) + \bar{p}_0 e^{-r(T-x)} (1 - F(T)) \right). \end{aligned}$$

In the benchmark infinite-lifetime case $T = \infty$, we can recover the buy-and-hold price from this fundamental valuation V by taking the maximum. Indeed, in the case $T = \infty$ all the discrete-trade buy-and-hold prices (3) together with their continuous-trade limit $\bar{p}_0$ are the same thing as the most optimistic fundamental valuation:

$$\bar{p}_0 = \max_{x \in [0, \infty)} V(x), \tag{6}$$

which in the infinite-lifetime literature is used as definition. The fundamental valuation, so extended, remains valid for both purposes: representing the buy-and-hold price (Proposition 4) and giving a sufficient condition for pricing at a resale premium. In representing the buy-and-hold price, the difference from the standard case is that the most optimistic fundamental valuation itself depends on the continuous-trade limit $\bar{p}_0$:

Proposition 4. *The buy-and-hold price's continuous-trade limit* $\bar{p}_0$ *equals the most optimistic fundamental valuation:*

$$\bar{p}_0 = \max_{x \in [0,T] \setminus \{\infty\}} V(x)$$

and the maximum exists.

We are also ready to extend to overlapping generations the idea of perpetual valuation switching from the infinite-lifetime models of Morris (1996) and Werner (2020), who find it to be sufficient for pricing at a resale premium. The requirement becomes that no investor assess from some age for the rest of life the fundamental value to be at least as high, conditional on the dividend remaining unpaid, as everyone else:

Definition 8 (Valuation-switching Condition)**.** The fundamental valuation V exhibits switching if some (relatively young) age $\hat{x} \in [0, T)$ is a maximizer of V but another (older) age $\breve{x} \in (\hat{x}, T)$ is not.

In other words, here investors switch from belonging to the group with the most optimistic fundamental valuation when they are of age $\hat{x}$ to a less optimistic group when they reach the age of $\breve{x}$, as long as the dividend is unpaid. This condition is sufficient for a nonzero resale premium in the continuous-trade limit as the discrete-trade equilibrium prices tend to the limit found in Proposition 3 and denoted by

$$p_0^* = \lim_{\Delta \to 0^+} p_\Delta^*,$$

that is:

Proposition 5 (Intergenerational Resale Premium 1). *If the fundamental valuation V exhibits switching, then $p_0^* > \bar{p}_0$.*

4.2.2. 2nd Sufficient Condition for Pricing at Resale Premium

Whereas the first sufficient condition (Definition 8) for a resale premium to appear is not directly on primitives of the model, our main condition, given here, is, but there is a striking similarity with the first condition. Our main condition requires something like valuation switching in the first condition, extended from Morris (1996) and Werner (2020), but requires it of the hazard rate instead of the fundamental valuation. Here is the main condition:

Definition 9 (End-of-life Hazard Switching). The hazard rate h exhibits end-of-life switching if

$$\lim_{x \to T^-} h(x) < \sup_{x \in (0,T)} h(x).$$

In other words, eventually, with age, the hazard rate h of the perceived remaining time W has to switch to and stay at lower levels relative to the highest ones, whereas the first condition did not require the fundamental valuation V to stay lower (Definition 8). Interpreting the hazard rate h as instantaneous optimism about the dividend (conditional on it remaining unpaid) as a function of age, the condition says that the most senior investors are not instantaneously the most optimistic. It is also sufficient for a resale premium to appear:

Proposition 6 (Intergenerational Resale Premium 2). *If the hazard rate h exhibits end-of-life switching, then $p_0^* > \bar{p}_0$.*

4.2.3. Converse: Necessary Condition for Pricing at Resale Premium

The remaining part of our characterization of buy-and-resell pricing at a resale premium is the necessity of our last sufficient condition. The equilibrium price coincides with the buy-and-hold price in the continuous-trade limit of our discrete-trade model if the sufficient condition—end-of-life hazard switching—fails:

Proposition 7 (No Intergenerational Resale Premium)**.** *If the hazard rate h does not exhibit end-of-life switching, then $p_0^* = \bar{p}_0$.*

Being the converse of Proposition 6, this result confirms that an intergenerational resale premium appears if and only if the most senior investors are not instantaneously the most optimistic, with instantaneous optimism being measured by the hazard rate h. It characterizes pricing at a resale premium even though the equilibrium and benchmark buy-and-hold prices can equal some long-term expected discounted returns of holding the asset. The latter reflect optimism more generally, over different holding durations, or trading strategies, which we speak of in interpreting the equilibrium price (Section 2), the buy-and-hold price (Section 3), and the first sufficient condition (Section 4.2.1).

4.3. Examples

We give three examples where our necessary and sufficient condition either fails so that Proposition 7 applies or holds so that Proposition 6 does. For convenience, we summarize them in Table 1, where, in particular, we indicate whether a nonzero resale premium and belief heterogeneity co-occur or only the latter occurs (the last column). In other words, our necessary and sufficient condition for a resale premium to appear is stronger than belief heterogeneity.

First of all, an example where our necessary and sufficient condition fails so that Proposition 7 applies is to let the perceived remaining time W be finite-valued with an exponential distribution and everything else be arbitrary (see Table 1). Indeed, in this case the hazard rate h is constant and consequently does not exhibit end-of-life switching, as desired.

To give an example where our necessary and sufficient condition holds so that Proposition 6 applies, let the perceived remaining time W have an incomplete exponential distribution, as in Example 1, and everything else be arbitrary (see Table 1). Indeed, in this case the hazard rate h is strictly decreasing and thus exhibits end-of-life switching, as required.

Finally, to see that our necessary and sufficient condition is indeed stronger than belief heterogeneity, let W be finite-valued and gamma-distributed with the shape parameter greater than one, $T < \infty$, and everything else be arbitrary (see Table 1). Indeed, in this case the hazard rate h is strictly increasing, i.e., more senior investors' beliefs are instantaneously more optimistic, but a resale premium does not appear (by Proposition 7).

5. Large Resale Premiums

The intergenerational resale premium is very sensitive to imperviousness of beliefs to information that is not experienced-based; the premium can be large even though histories which most investors live through differ only in the birth date. In fact, the equilibrium price can be very high not only relative to the buy-and-hold price, but its continuous-trade limit can hit an "upper bound [of $1] on any reasonable fundamental value" as under asymmetric information in Allen et al. (1993). The upper bound of $1 comes from the fact that the only dividend is $1 at the uncertain terminal time. The equilibrium price is as high as this in the continuous-trade limit when newborn investors' beliefs are instantaneously sufficiently optimistic in the sense that the hazard rate h diverges to infinity:

Table 1: Characterization of Pricing at Resale Premium Applied to Some Distributions

	Exponential	Incomplete exponential	Gamma(α, λ), $\alpha > 1$
Parameters	$\lambda \in (0, \infty)$	$\lambda \in (0, \infty)$, $q \in (0, 1)$	$\lambda \in (0, \infty)$, $\alpha \in (1, \infty)$
$F(w)$, $w \geq 0$	$1 - e^{-\lambda w}$	$(1 - q)\left(1 - e^{-\lambda w}\right)$	$\frac{\lambda^{\alpha}}{\Gamma(\alpha)} \int_0^w z^{\alpha - 1} e^{-\lambda z} dz$
$f(w)$, $w > 0$	$\lambda e^{-\lambda w}$	$(1 - q) \lambda e^{-\lambda w}$	$\frac{\lambda^{\alpha}}{\Gamma(\alpha)} w^{\alpha - 1} e^{-\lambda w}$
$h(x)$, $0 < x < T$	λ	$\left(1 - \frac{q}{q + (1 - q) e^{-\lambda x}}\right) \lambda$	$\frac{1}{\int_0^{\infty} \left(1 + \frac{w}{x}\right)^{\alpha - 1} e^{-\lambda w} dw}$
Hazard-rate shape	Constant	Strictly decreasing	Strictly increasing
End-of-life hazard switching	No	Yes	No
Nonzero Resale Premium	*No*	*Yes*	*No*
$F(w\|x)$, $w \geq 0$, $0 < x < T$	$F(w)$	$\frac{F(w) h(x)}{(1 - q) \lambda}$	$\frac{F(w + x) - F(x)}{1 - F(x)}$
$\frac{\partial}{\partial x} F(w\|x)$, $w > 0$, $0 < x < T$	0	$\frac{F(w) h'(x)}{(1 - q) \lambda} < 0$	$(1 - F(w\|x))$ $\times (h(w + x) - h(w))$ > 0
Belief heterogeneity (across x)	No	Yes	Yes
$F(\cdot\|x)$ FOSD $F(\cdot\|x')$, $x, x' \in [0, T)$	$F(\cdot\|x) = F(\cdot\|x')$	Iff $x \geq x'$	Iff $x \leq x'$
Assumptions throughout paper	All hold	All hold	All hold if $T < \infty$, Assumption 3 fails if $T = \infty$

Proposition 8 (Pricing at No Fundamental Value). *If* $\lim_{x \to 0^+} h(x) = \infty$, *then* $p_0^* = 1$.

In this proposition, the equilibrium price's continuous-trade limit hits the bound of what one can get by taking expected discounted values of dividends for all possible probabilities. The equilibrium price as high as this is as if the one-shot dividend of $1 were imminent and there were no uncertainty about its timing. This already shows that apparently innocuous differences in histories which investors live through can push the equilibrium price as high as any reasonable fundamental value could possibly be. Our remaining question is how high the equilibrium price can be in the continuous-trade limit relative to the buy-and-hold price, which is the most optimistic fundamental valuation across coexisting investors within the model (Proposition 4). We can find the buy-and-hold price if we know a maximizer of the fundamental valuation V and this maximizer is not the terminal age T, which is always a maximizer in the finite-lifetime case $T < \infty$ by Definition 7 and Proposition 4. Additional maximizers, if any, are likely to be close to where the hazard rate h is the highest, as the following proposition shows for the case of h eventually decreasing:

Proposition 9 (Most Optimistic Fundamental Valuation at Early Age). *If the hazard rate* h *is (weakly) decreasing on* (x', T) *for some* $x' \in (0, T)$, *then at least some maximizers of the fundamental valuation* V *belong to* $[0, x']$.

When, in particular, younger investors are instantaneously equally or more optimistic than older ones in the sense that the hazard rate h is decreasing, newborn investors' fundamental valuation $V(0)$ is at least as high as everyone else's. An explicit expression for the buy-and-hold price's continuous-trade limit $\bar{p}_0$ now follows by the definition of fundamental valuation (Definition 7):

Corollary 1 (Most Optimistic Fundamental Valuation at Birth). *If the hazard rate* h *is (weakly) decreasing, then the age of 0 is a maximizer of the fundamental valuation* V

with

$$\bar{p}_0 = V(0) = \frac{\int_0^T e^{-rw} dF(w)}{1 - e^{-rT}(1 - F(T))}.$$

This corollary and Proposition 8 give conditions under which an explicit comparison of the equilibrium and buy-and-hold prices in the continuous-trade limit is possible. An example where both conditions hold is to let the perceived remaining time W be finite-valued and gamma-distributed with the shape parameter less than one and everything else be arbitrary:

Example 2 (Large Resale Premiums under Gamma Distribution)**.** Suppose that W is finite-valued and $W \sim \text{Gamma}(\alpha, \lambda)$ with $\alpha < 1$, which does not change the formulas for the distribution F, the density f, and the hazard rate h given in Table 1. By Proposition 8, Corollary 1, and Proposition 6, we have

$$p_0^* = 1 > \frac{\lambda^\alpha}{\Gamma(\alpha)} \frac{\int_0^T e^{-rw} w^{\alpha-1} e^{-\lambda w} dw}{1 - e^{-rT}(1 - F(T))} = \bar{p}_0. \tag{7}$$

Consider the equilibrium and buy-and-hold prices' continuous-trade limits p_0^* and $\bar{p}_0$ as (real) functions of the scale parameter λ on $(0, \infty)$. While the former, on the left-hand side of (7), is constant at \$1, the latter becomes zero in the limit

$$\lim_{\lambda \to 0^+} \bar{p}_0(\lambda) = 0,$$

hence the intergenerational resale premium can be arbitrarily large in the sense that

$$\lim_{\lambda \to 0^+} \frac{p_0^*(\lambda) - \bar{p}_0(\lambda)}{\bar{p}_0(\lambda)} = \infty.$$

6. Conclusion

A documented experience effect—imperviousness to information that is not experience-based—can in the absence of short-selling push the equilibrium price of the risky asset as high as any reasonable fundamental value could possibly be. The buy-and-hold price—the highest fundamental valuation among investors—can be arbitrarily low, implying frequent retrading at an arbitrarily large intergenerational resale premium.

A. Proof of Propositions 2–9

The proofs of individual propositions come in separate subsections of a single proof, meaning that in any subsection we are free to cross-reference to steps made in any of the preceding subsections. Keeping track of assumptions does not become a problem, because each assumption, once stated, applies for the rest of the paper.

Proposition 2

We unify parts (i) and (ii) into one problem by considering a general form of the constraint on holding durations. In both cases, this constraint has the form of a correspondence Z_Δ from the participating investors' age interval $[0, T - \Delta] \backslash \{\infty\}$ to the holding durations $\{\Delta, 2\Delta, 3\Delta, \dots, \infty\}$ defined by either

$$Z_\Delta(x) = Y_\Delta(x), \text{ for part (ii)},$$

or

$$Z_\Delta(x) = \{\Delta, 2\Delta, 3\Delta, \dots, \infty\} \cap [\Delta, T - x], \text{ for part (i)}.$$

Now the unified problem is to find a unique price $p \in [0,1]$ such that

$$p = \max_{\substack{x \in [0, T-\Delta] \setminus \{\infty\} \\ y \in Z_\Delta(x)}} \mathrm{E}\left(e^{-r(W-x)} I_{\{W \leq x+y\}} + p e^{-ry} I_{\{W > x+y\}} \middle| W > x\right) \tag{8}$$

and the maximum exists. Such a price p is either an equilibrium price (part (i)) or a buy-and-hold price (part (ii)) depending on the choice of Z_Δ, and thus it suffices to solve the unified problem (8). It is a combination of a maximization problem (noncompact if $T = \infty$) and a fixed-point problem. For the noncompact maximization to work we had simply included and will use the assumption of eventual first-order stochastic dominance (Assumption 3), and for the fixed point we will use the Contraction-mapping Theorem.

For solving the unified problem (8) as just outlined, it is first convenient to write the maximand expectation in (8) in terms of the distribution and in two different ways (9) and (10):

$$\begin{aligned}
&\mathrm{E}\left(e^{-r(W-x)} I_{\{W \leq x+y\}} + p e^{-ry} I_{\{W > x+y\}} \middle| W > x\right) \\
&\quad = \mathrm{E}\left(e^{-r(W-x)} I_{\{W-x \leq y\}} + p e^{-ry} I_{\{W-x > y\}} \middle| W > x\right) \\
&\quad = \int_{-\infty}^{y} e^{-rw} dF\left(w|x\right) + \int_{y}^{\infty} p e^{-ry} dF\left(w|x\right) + p e^{-ry}\left(1 - \lim_{w \to \infty} F\left(w|x\right)\right) \qquad (9) \\
&\quad = \frac{1}{1-F\left(x\right)}\left(\int_{x}^{x+y} e^{-r(w-x)} dF\left(w\right) + p e^{-ry}\left(1 - F\left(x+y\right)\right)\right), \qquad (10)
\end{aligned}$$

for each age $x \in [0, T-\Delta] \setminus \{\infty\}$ and for each holding duration $y \in Z_\Delta(x)$. By the former formula (9) and Assumption 3, either $T < \infty$ and the maximization is over the compact subset

$$\left\{(x,y) \in \mathbb{R}^2 : x \in [0, T-\Delta], y \in Z_\Delta(x)\right\}$$

of $\mathbb{R}^2$ or else the maximum coincides (either both or neither exist) with the maximum over the compact subset

$$[0, \tilde{x}] \times Z_{\Delta}(0) \tag{11}$$

of $\mathbb{R}$ times its one-point compactification $(-\infty, \infty]$ (see, e.g., Aliprantis and Border, 2006). From the latter formula (10) and Assumption 2, it is easy to see continuity of the objective function on these compact sets, implying the existence of the maximum, which we denote by $v_{\Delta}(p)$, for each price $p \in [0, 1]$. This reduces the whole problem (8) to finding a unique fixed point $p \in [0, 1]$ of this function $v_{\Delta} : [0, 1] \to [0, 1]$. For this final step, it suffices to show that this v_{Δ} is a contraction, and thus it is enough to show that all prices $p, p' \in [0, 1]$ with $p \geq p'$ satisfy

$$|v_{\Delta}(p) - v_{\Delta}(p')| \leq e^{-r\Delta} |p - p'| .$$

To verify this inequality, we use any maximizer (x, y) of the right-hand side of (8) when the price is p and directly calculate that

$$\begin{aligned}
|v_{\Delta}(p) - v_{\Delta}(p')| &= v_{\Delta}(p) - v_{\Delta}(p') \\
&= \frac{1}{1 - F(x)} \left(\int_{x}^{x+y} e^{-r(w-x)} dF(w) + pe^{-ry} (1 - F(x+y)) \right) - v_{\Delta}(p') \\
&\leq \frac{1}{1 - F(x)} \left(pe^{-ry} (1 - F(x+y)) - p' e^{-ry} (1 - F(x+y)) \right) \\
&= \frac{1 - F(x+y)}{1 - F(x)} e^{-ry} (p - p') \\
&\leq e^{-ry} (p - p') \leq e^{-r\Delta} (p - p') = e^{-r\Delta} |p - p'| ,
\end{aligned}$$

completing the proof of Proposition 2.

Proposition 3

We consider the unique solution p_Δ of the unified problem (8) as a (real) function of the trade-frequency parameter Δ on $(0,T)$ and show that this function has a right-hand limit at 0 in four steps: characterizing the fixed points p_Δ of the maxima $v_\Delta : [0,1] \to [0,1]$ as maxima of fixed points, extending these new maximands to continuous trade, extending the constraints on holding durations to continuous trade, and then obtaining the desired limit using the Berge Maximum Theorem. This continuous-trade limit applies to either the equilibrium price (part (i)) or buy-and-hold price (part (ii)), because we view each of them as the solution of the unified problem (8).

Step 1. Characterizations of Discrete-trade Solutions

We simplify the mathematical structure of the definition of the solution p_Δ of the unified discrete-trade problem (8) by two characterizations (12) and (13) below. The second characterization is just a useful improvement of the first.

First to characterize the discrete-trade solution p_Δ by showing that

$$p_\Delta = \max_{\substack{x \in [0,T-\Delta] \setminus \{\infty\} \\ y \in Z_\Delta(x)}} \frac{\int_x^{x+y} e^{-r(w-x)} dF(w)}{1 - F(x) - e^{-ry}(1 - F(x+y))} \tag{12}$$

and the maximum exists, where the maximand is the fixed point of the original price-dependent maximand in the discrete-trade problem (8). This holds for the solution p_Δ because its definition (8) requires that all $x \in [0, T-\Delta] \setminus \{\infty\}$ and all $y \in Z_\Delta(x)$ satisfy

$$p_\Delta \geq \frac{1}{1-F(x)} \left(\int_x^{x+y} e^{-r(w-x)} dF(w) + p_\Delta e^{-ry} (1 - F(x+y)) \right),$$

hence

$$p_\Delta \geq \frac{\int_x^{x+y} e^{-r(w-x)} dF(w)}{1 - F(x) - e^{-ry}(1 - F(x+y))},$$

but both inequalities become equalities when (x, y) is any of the original maximizers in (8).

Now a useful improvement of this characterization (12) of the solution p_Δ follows by restricting the maximization there to a compact set in the infinite-lifetime case $T = \infty$ like in the earlier characterization, the one using (11). To write down this improved characterization in a unified way for both the infinite- and finite-lifetime cases, we denote the right endpoint of the age interval used by x_Δ: x_Δ is $\tilde{x}$ from Assumption 3 or $T - \Delta$ according as $T = \infty$ or $T < \infty$. The useful fact we were after is that

$$p_\Delta = \max_{\substack{x \in [0, x_\Delta] \\ y \in Z_\Delta(x)}} \frac{\int_x^{x+y} e^{-r(w-x)} dF(w)}{1 - F(x) - e^{-ry}(1 - F(x+y))} \tag{13}$$

and follows in the same way as the first version (12), except that in the discrete-trade problem (8) we take a maximizer belonging to this smaller set using again the maximization over (11).

Step 2. Continuous Extension of Maximand from Step 1

To take the desired limit using the characterizations (12)–(13), we first show that the maximands there extend to a continuous real function g on

$$\{(x, y) \in \mathbb{R} \times (-\infty, \infty] : 0 \leq x \leq T, 0 \leq y \leq T - x\}, \tag{14}$$

where $(-\infty,\infty]$ is the one-point compactification of $\mathbb{R}$ (the unclear part is $y=0$). For this, we define g by

$$g(x,y)=\begin{cases}\int_x^{x+y} e^{-r(w-x)}dF(w)\Big/\left(1-F(x)-e^{-ry}\left(1-F(x+y)\right)\right) & \text{if } y>0,\\ \tilde{h}(x) & \text{if } y=0<x<T,\\ \lim_{w\to T^-}\tilde{h}(w) & \text{if } y=0<x=T,\\ \lim_{w\to 0^+}\tilde{h}(w) & \text{if } y=0=x,\end{cases}$$

so that it only remains to prove continuity of g, the crux of which is the L'Hôpital Rule that works in some sense uniformly, but for which we do not have an off-the-shelf statement and do our own proof. Namely, the key to continuity is to first prove that uniformly on compact subsets of $[0,T]\setminus\{\infty\}\subset\mathbb{R}$ we have

$$\lim_{y\to 0^+} g(x,y)=g(x,0), \tag{15}$$

where allowed y's depend on x, i.e., by (15) we mean that for every compact $X\subset[0,T]\setminus\{\infty\}$ and for every $\varepsilon\in(0,\infty)$ there is a $\delta\in(0,\infty)$ such that all $x\in X$ and all $y\in[0,T-x]\cap(0,\delta)$ satisfy

$$|g(x,y)-g(x,0)|<\varepsilon. \tag{16}$$

Let us prove this key claim (15) and then return to continuity of g. Since in this claim any $\delta\in(0,\infty)$ works if $x=T$, we only need to find a $\delta\in(0,\infty)$ that works for every $x\in X\backslash\{T\}$. We note that $x<T$ and so for all $y,y'\in(0,T-x)$ with $y>y'$ the Cauchy Mean-value Theorem, whose differentiability hypotheses follow from Assumption 5 by the Fundamental Theorem of Calculus, yields a $z \in (y', y)$ such that

$$\begin{aligned}
&\frac{\int_x^{x+y} e^{-r(w-x)} dF(w) - \int_x^{x+y'} e^{-r(w-x)} dF(w)}{1 - F(x) - e^{-ry}(1 - F(x+y)) - (1 - F(x) - e^{-ry'}(1 - F(x+y')))} \\
&= \frac{e^{-rz} f(x+z)}{e^{-rz}(1 - F(x+z)) r + e^{-rz} f(x+z)} \\
&= \frac{f(x+z)}{f(x+z) + (1 - F(x+z)) r} \\
&= \frac{h(x+z)}{h(x+z) + r} \\
&= \tilde{h}(x+z) \\
&= g(x+z, 0).
\end{aligned}$$

Now since the function $g(\cdot, 0)$ on $[0, T] \setminus \{\infty\}$ is continuous considering continuity of the density f as per Assumption 5, this very $g(\cdot, 0)$ is uniformly continuous on compact subsets of $[0, T] \setminus \{\infty\}$. This means that either $T < \infty$ and $g(\cdot, 0)$ is uniformly continuous or else $T = \infty$ and $g(\cdot, 0)$ is uniformly continuous on $X + [0, 1]$. In both cases, uniform continuity yields a $\delta \in (0, \infty)$ independent of x such that

$$\begin{aligned}
&y \in (0, \delta) \implies \\
&|g(x+z, 0) - g(x, 0)| < \frac{\varepsilon}{2} \implies \\
&\left| \frac{\int_x^{x+y} e^{-r(w-x)} dF(w) - \int_x^{x+y'} e^{-r(w-x)} dF(w)}{1 - F(x) - e^{-ry}(1 - F(x+y)) - (1 - F(x) - e^{-ry'}(1 - F(x+y')))} - g(x, 0) \right| < \frac{\varepsilon}{2}.
\end{aligned}$$

Passing to the limit as $y' \to 0^+$ shows that

$$|g(x, y) - g(x, 0)| \leq \frac{\varepsilon}{2}.$$

Since here y was an arbitrary element of $(0, T - x) \cap (0, \delta)$, this desired inequality readily extends to all $y \in [0, T - x] \cap (0, \delta)$. Since here x was an arbitrary element of $X \setminus \{T\}$, we have proven the desired uniform convergence stated in (15).

It remains to verify continuity of g now that we have its convergence (15), as $y \to 0^+$, uniformly on compact subsets of $[0,T] \setminus \{\infty\}$ in the sense of (16). For continuity, it is convenient to use the sequential criterion, but we only look at sequences $\{(x_n, y_n)\}$ in the domain (14) of g converging to (x,y) in its domain with $y = 0$, as the rest are rudimental. It is precisely the former cases where knowing the uniform-convergence property (15) helps, because $\{(x_n, y_n)\} \to (x, 0)$ means that the set $\{x, x_1, x_2, \dots\}$ is a compact subset of $[0,T] \setminus \{\infty\}$ and we have that uniform convergence. Now to complete this step, note that by this property for every $\varepsilon \in (0,\infty)$ there is a $\delta_2 \in (0,\infty)$ such that all indices n satisfy

$$y_n \in (0, \delta_2) \implies |g(x_n, y_n) - g(x_n, 0)| < \frac{\varepsilon}{2}$$

and (recall continuity of $g(\cdot, 0)$ from the previous paragraph) there is a $\delta_1 \in (0,\infty)$ such that all n satisfy

$$|x_n - x| < \delta_1 \implies |g(x_n, 0) - g(x, 0)| < \frac{\varepsilon}{2},$$

hence eventually

$$|g(x_n, y_n) - g(x, 0)| < \varepsilon,$$

as desired.

Step 3. Continuous-trade Holding Constraints

To furnish a candidate for the desired limit of the discrete-trade solution based on its characterizations (12)–(13) from Step 1, we also take their constraint on holding durations and extend it to continuous trade. Our new constraint is the correspondence

Z_0 from the coexisting investors' age interval $[0,T]\setminus\{\infty\}$ to the holding durations $[0,\infty]$ defined by either

$$Z_0(x) = \{T-x\}, \text{ for part (ii)}, \tag{17}$$

or

$$Z_0(x) = [0, T-x], \text{ for part (i)}. \tag{18}$$

In its domain, define x_0 to be $\tilde{x}$ from Assumption 3 or T according as $T=\infty$ or $T<\infty$.

Step 4. Convergence of Discrete-trade Solutions

The final step is to see from Steps 1–3 that the desired limits are (in unified form for both part (i) and (ii) by means of notation (17)–(18) for the constraint on holding durations)

$$\lim_{\Delta\to 0^+} p_\Delta = \max_{\substack{x\in[0,x_0] \\ y\in Z_0(x)}} g(x,y). \tag{19}$$

Here we only note that what we are taking the limit of are themselves maxima (13) and they converge to this maximum by a version of the Berge Maximum Theorem that only assumes continuity of the correspondence at a single point (see Moore, 2010).

Proposition 4

Observation (6) settles the infinite-lifetime case $T=\infty$. For the remaining finite-lifetime scenario $T<\infty$, we note that formula (19) requires at every age $x\in[0,T)$ that

$$\bar{p}_0 \geq g(x, T-x) = \frac{\int_x^T e^{-r(w-x)} dF(w)}{1-F(x)-e^{-r(T-x)}(1-F(T))}, \tag{20}$$

hence

$$\bar{p}_0 \geq \frac{1}{1-F(x)}\left(\int_x^T e^{-r(w-x)}dF(w) + \bar{p}_0 e^{-r(T-x)}(1-F(T))\right) = V(x), \tag{21}$$

but $\bar{p}_0 = V(T)$, readily completing the proof of Proposition 4.

Proposition 5

First we take $\hat{x}, \breve{x}$ from Definition 8 and, without loss of generality, $\hat{x} \in [0, \tilde{x}]$ if $T = \infty$ (maximizers of (10), where p drops out, over (11) for the case of buy-and-hold price (part (ii)), where Δ drops out, give us maximizers of V that belong to $[0, \tilde{x}]$). Now the proof goes by showing two inequalities

$$p_0^* \geq g(\hat{x}, \breve{x} - \hat{x}) > \bar{p}_0, \tag{22}$$

the first of which is immediate from (19):

$$p_0^* = \lim_{\Delta \to 0^+} p_\Delta^* = \max_{\substack{x \in [0, x_0] \\ y \in [0, T-x]}} g(x, y) \geq g(\hat{x}, \breve{x} - \hat{x}).$$

For the remaining inequality (the strict one in (22)), first note that $V(\hat{x}) = \bar{p}_0$ by Proposition 4, but also

$$V(\hat{x}) = \frac{\int_{\hat{x}}^{\breve{x}} e^{-r(w-\hat{x})}dF(w) + \bar{p}_0 e^{-r(\breve{x}-\hat{x})}(1-F(\breve{x}))}{1-F(\hat{x})} + \frac{1-F(\breve{x})}{1-F(\hat{x})}e^{-r(\breve{x}-\hat{x})}(V(\breve{x}) - \bar{p}_0)$$

by a direct calculation. It follows from these two conditions that

$$\begin{aligned} \frac{\int_{\hat{x}}^{\breve{x}} e^{-r(w-\hat{x})}dF(w) + \bar{p}_0 e^{-r(\breve{x}-\hat{x})}(1-F(\breve{x}))}{1-F(\hat{x})} &= V(\hat{x}) + \frac{1-F(\breve{x})}{1-F(\hat{x})}e^{-r(\breve{x}-\hat{x})}(\bar{p}_0 - V(\breve{x})) \\ &= \bar{p}_0 + \frac{1-F(\breve{x})}{1-F(\hat{x})}e^{-r(\breve{x}-\hat{x})}(V(\hat{x}) - V(\breve{x})). \end{aligned}$$

Now the time is ripe for applying valuation switching, by which here $V(\hat{x}) - V(\breve{x}) > 0$, to conclude that

$$\frac{1}{1 - F(\hat{x})} \left(\int_{\hat{x}}^{\breve{x}} e^{-r(w-\hat{x})} dF(w) + \bar{p}_0 e^{-r(\breve{x}-\hat{x})} (1 - F(\breve{x})) \right) > \bar{p}_0,$$

and thus that

$$\bar{p}_0 < \frac{\int_{\hat{x}}^{\breve{x}} e^{-r(w-\hat{x})} dF(w)}{1 - F(\hat{x}) - e^{-r(\breve{x}-\hat{x})} (1 - F(\breve{x}))} = g(\hat{x}, \breve{x} - \hat{x}),$$

as desired.

Proposition 6

Taking advantage of Proposition 5, we assume for this proof that the fundamental valuation V does not exhibit switching. With this, the proof goes by showing two things

$$p_0^* > \lim_{x \to T^-} \tilde{h}(x) = \bar{p}_0, \tag{23}$$

but the inequality is almost immediate from end-of-life hazard switching, by which there exists an $\hat{x} \in (0, T)$ such that

$$\lim_{x \to T^-} h(x) < h(\hat{x}),$$

and the fact that p_0^* is the limit in (19) of the discrete-trade equilibrium prices with representation (12):

$$\begin{aligned} p_0^* = \lim_{\Delta \to 0^+} p_\Delta^* &\geq \lim_{\Delta \to 0^+} g(\hat{x}, \Delta) \qquad\qquad (\Delta \in (0, T - \hat{x})) \\ &= g(\hat{x}, 0) \end{aligned}$$

$$
\begin{aligned}
&= \tilde{h}(\hat{x}) \\
&= \frac{h(\hat{x})}{h(\hat{x}) + r} \\
&> \frac{\lim_{x \to T^-} h(x)}{r + \lim_{x \to T^-} h(x)} \\
&= \lim_{x \to T^-} \tilde{h}(x).
\end{aligned}
$$

For the remaining part of the proof (the equality in (23)), we use the fact that we could assume that V does not exhibit switching and consider the following two cases.

Case 1. $T < \infty$

By contradiction, suppose that $\lim_{x \to T^-} \tilde{h}(x) \neq \bar{p}_0$. It follows that

$$
\lim_{x \to T^-} \tilde{h}(x) = g(T, 0) < \bar{p}_0 = \max_{x \in [0,T]} g(x, T - x)
$$

from (19), and thus that here some $\hat{x} \in [0, T)$ is a maximizer but some $\breve{x} \in (\hat{x}, T)$ is not. The next step is to conclude that $V(\hat{x}) = \bar{p}_0 > V(\breve{x})$ by noting that in (20)–(21) both inequalities become equalities or strict inequalities according as we plug $\hat{x}$ or $\breve{x}$. Now $\hat{x}$ maximizes V by Proposition 4 but $\breve{x}$ does not, and thus V exhibits switching, contradicting our assumption for the whole proof of Proposition 6.

Case 2. $T = \infty$

This goes by showing that V and $\tilde{h}$ are eventually constant at $\bar{p}_0$. First for V, take any of its maximizers $\hat{x}$, which exists with $V(\hat{x}) = \bar{p}_0$ by Proposition 4, and note that now all $x \in [\hat{x}, \infty)$ satisfy $V(x) = V(\hat{x}) = \bar{p}_0$ by our assumption for the whole proof of Proposition 6. Now for $\tilde{h}$, differentiate V on $(\hat{x}, \infty)$ to see that every $x \in (\hat{x}, \infty)$ satisfies

$$
0 = V'(x) = (h(x) + r) V(x) - h(x) = (h(x) + r) \bar{p}_0 - h(x),
$$

hence

$$\tilde{h}(x) = \frac{h(x)}{h(x) + r} = \bar{p}_0,$$

as desired.

Proposition 7

First note that it suffices to show that $p_0^* \leq \bar{p}_0$, because the reverse inequality holds by (19):

$$p_0^* = \max_{\substack{x \in [0, x_0] \\ y \in [0, T-x]}} g(x, y) \geq \max_{\substack{x \in [0, x_0] \\ y \in \{T-x\}}} g(x, y) = \bar{p}_0. \tag{24}$$

Now the proof goes by taking any maximizer (x^*, y^*) on the left-hand side of this inequality and showing three more inequalities

$$p_0^* = g(x^*, y^*) \leq g(x^*, 0) \leq \lim_{x \to T^-} \tilde{h}(x) \leq \bar{p}_0. \tag{25}$$

Among these, the middle inequality follows from the absence of end-of-life hazard switching by noting that

$$\lim_{x \to T^-} \tilde{h}(x) = \lim_{x \to T^-} \frac{h(x)}{h(x) + r} \geq \sup_{x \in (0, T)} \frac{h(x)}{h(x) + r} = \sup_{x \in (0, T)} g(x, 0) \geq g(x^*, 0),$$

the last inequality, in (25), holds because either $T < \infty$ and

$$\lim_{x \to T^-} \tilde{h}(x) = g(T, 0) \leq \max_{x \in [0, T]} g(x, T - x) = \bar{p}_0 \tag{26}$$

again by (19) or else $T = \infty$ and

$$\begin{aligned}
\lim_{x\to\infty} \tilde{h}(x) &= \lim_{x\to\infty} \frac{h(x)}{h(x)+r} && (27)\\
&= \lim_{x\to\infty} \frac{\int_x^\infty e^{-rw} dF(w)}{e^{-rx}(1-F(x))} && \text{(by the L'Hôpital Rule)}\\
&= \lim_{x\to\infty} \frac{1}{1-F(x)} \int_x^\infty e^{-r(w-x)} dF(w)\\
&= \lim_{x\to\infty} V(x)\\
&\le \max_{x\in[0,\infty)} V(x)\\
&= \bar{p}_0 && \text{(by (6))},
\end{aligned}$$

and to prove the first inequality, in (25), consider the following three cases.

Case 1. $x^* = 0$

By contradiction, suppose that $g(x^*, y^*) > g(x^*, 0)$. It follows by definitions that

$$p_0^* > \lim_{x\to 0^+} \tilde{h}(x). \tag{28}$$

Now it must be that $\lim_{x\to 0^+} \tilde{h}(x) < 1$, because $p_0^* = \lim_{\Delta\to 0^+} p_\Delta^* \le 1$. It then follows that $\lim_{x\to 0^+} h(x) < \infty$. Furthermore, f has the same limit

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} h(x) < \infty, \tag{29}$$

which is to say $\lim_{x\to 0^+} f(x) < \infty$. Next as a density for F also define $f_0 : \mathbb{R} \to [0,\infty)$ by

$$f_0(w) = \begin{cases} \lim_{x\to 0^+} f(x) & \text{if } w = 0,\\ f(w) & \text{if } w \ne 0. \end{cases}$$

This way, f_0 is continuous on $[0,T)$ by Assumption 5. Now the Fundamental Theorem of Calculus allows us to differentiate $v:[0,y^*)\to\mathbb{R}$ defined by

$$v(x)=g(x,y^*-x)=\frac{\int_x^{y^*}e^{-r(w-x)}dF(w)}{1-F(x)-e^{-r(y^*-x)}(1-F(y^*))} \tag{30}$$

to see that its derivative at every $x\in[0,y^*)$ is

$$v'(x)=\frac{(f_0(x)+r(1-F(x)))\,v(x)-f_0(x)}{1-F(x)-e^{-r(y^*-x)}(1-F(y^*))}.$$

Furthermore, we have

$$v'(0)=\frac{(\lim_{x\to0^+}h(x)+r)\,p_0^*-\lim_{x\to0^+}h(x)}{1-e^{-ry^*}(1-F(y^*))}>0,$$

because (28) and (29) show that

$$p_0^*>\lim_{x\to0^+}\tilde{h}(x)=\frac{\lim_{x\to0^+}h(x)}{r+\lim_{x\to0^+}h(x)}.$$

It follows that some $\hat{x}\in(0,y^*)$ satisfies $v(\hat{x})>v(0)=p_0^*$. Finally, for the contradiction, let Δ be $y^*-\hat{x}$ or 1 according as $y^*<\infty$ or $y^*=\infty$, let p_Δ^* be the corresponding (discrete-trade) equilibrium price found in Proposition 2, and observe, using (12) and (13), that

$$\begin{aligned} p_0^* &< v(\hat{x})=g(\hat{x},y^*-\hat{x}) \\ &\le \max_{\substack{x\in[0,T-\Delta]\setminus\{\infty\}\\ y\in\{\Delta,2\Delta,3\Delta,\ldots,\infty\}\cap[\Delta,T-x]}} g(x,y)=p_\Delta^*=\max_{\substack{x\in[0,x_\Delta]\\ y\in\{\Delta,2\Delta,3\Delta,\ldots,\infty\}\cap[\Delta,T-x]}} g(x,y) \\ &\le \max_{\substack{x\in[0,x_0]\\ y\in[0,T-x]}} g(x,y), \end{aligned} \tag{31}$$

contradicting the formula (used in (24)) for p_0^*.

Case 2. $0 < x^* < T$

It goes either trivially if $y^* = 0$ or by taking a first-order necessary condition for an interior maximum in the following way if $y^* > 0$. First of all, as an objective function it is convenient to consider $v : (0, x^* + y^*) \to \mathbb{R}$ defined by

$$v(x) = g(x, x^* + y^* - x) = \frac{\int_x^{x^*+y^*} e^{-r(w-x)} dF(w)}{1 - F(x) - e^{-r(x^*+y^*-x)}(1 - F(x^* + y^*))} \tag{32}$$

similarly to (30). Now we show that x^* maximizes v by adjusting (31): let $x' \in (0, x^* + y^*)$, let Δ be $x^* + y^* - x'$ or 1 according as $y^* < \infty$ or $y^* = \infty$, let p^*_Δ be the corresponding (discrete-trade) equilibrium price found in Proposition 2, and observe, using (12) and (13), that

$$\begin{aligned} v(x') &= g(x', x^* + y^* - x') \\ &\leq \max_{\substack{x \in [0, T-\Delta] \setminus \{\infty\} \\ y \in \{\Delta, 2\Delta, 3\Delta, \ldots, \infty\} \cap [\Delta, T-x]}} g(x, y) = p^*_\Delta = \max_{\substack{x \in [0, x_\Delta] \\ y \in \{\Delta, 2\Delta, 3\Delta, \ldots, \infty\} \cap [\Delta, T-x]}} g(x, y) \\ &\leq \max_{\substack{x \in [0, x_0] \\ y \in [0, T-x]}} g(x, y) = g(x^*, y^*) = g(x^*, x^* + y^* - x^*) \\ &= v(x^*). \end{aligned}$$

Next differentiate v at x^* to see that

$$0 = v'(x^*) = \frac{(f(x^*) + r(1 - F(x^*)))\, g(x^*, y^*) - f(x^*)}{1 - F(x^*) - e^{-ry^*}(1 - F(x^* + y^*))},$$

hence

$$g(x^*, y^*) = \frac{f(x^*)}{f(x^*) + r(1 - F(x^*))} = \frac{h(x^*)}{h(x^*) + r} = \tilde{h}(x^*) = g(x^*, 0),$$

proving the first inequality in (25), as desired, for this case.

Case 3. $x^* = T$

It is immediate from (24) because $y^* = 0$ by the constraints, completing the proof of Proposition 7.

Proposition 8

We use formula (19) and calculate that

$$1 \geq \lim_{\Delta \to 0^+} p^*_\Delta = p^*_0 = \max_{\substack{x \in [0,x_0] \\ y \in [0, T-x]}} g(x,y) \geq g(0,0) = \lim_{x \to 0^+} \tilde{h}(x) = \frac{\lim_{x \to 0^+} h(x)}{r + \lim_{x \to 0^+} h(x)} = 1.$$

These inequalities readily complete the proof of Proposition 8.

Proposition 9

First we define $v : [0, T) \to \mathbb{R}$ by

$$v(x) = g(x, T-x) = \frac{\int_x^T e^{-r(w-x)} dF(w)}{1 - F(x) - e^{-r(T-x)}(1 - F(T))}$$

similarly to (30) and (32), note that its derivative at every $x \in (0, T)$ is

$$v'(x) = \frac{(f(x) + r(1 - F(x)))\, v(x) - f(x)}{1 - F(x) - e^{-r(T-x)}(1 - F(T))}, \tag{33}$$

and take the limit

$$\lim_{x \to T^-} v(x) = \lim_{x \to T^-} \tilde{h}(x) \tag{34}$$

already seen in (26) and (27). Next Step 1 shows that maximizers of v are also maximizers of V, Step 2 establishes that v is either decreasing on (x', T) or strictly increasing on $[x'', T)$ for some $x'' \in (x', T)$, and, finally, Step 3 eliminates the latter alternative.

Step 1. Reduction to v

If $T = \infty$, then $V = v$. If $T < \infty$ and some $x^* \in [0, T)$ maximizes v, then x^* maximizes V because (19), where g is continuous, implies that

$$\frac{\int_{x^*}^{T} e^{-r(w-x^*)} dF(w)}{1 - F(x^*) - e^{-r(T-x^*)}(1 - F(T))} = v(x^*) = \max_{x \in [0,T)} v(x) = \max_{x \in [0,T)} g(x, T - x)$$
$$= \max_{x \in [0,T]} g(x, T - x) = \bar{p}_0,$$

hence in view of Proposition 4 we have

$$V(x^*) = \frac{1}{1 - F(x^*)} \left(\int_{x^*}^{T} e^{-r(w-x^*)} dF(w) + \bar{p}_0 e^{-r(T-x^*)}(1 - F(T)) \right) = \bar{p}_0$$
$$= \max_{x \in [0,T]} V(x).$$

Step 2. Two Monotonicity Scenarios

By contradiction, suppose that v is neither decreasing on (x', T) nor strictly increasing on $[x'', T)$ for any $x'' \in (x', T)$. This means, in view of (33), that some $x \in (x', T)$ satisfies $v'(x) > 0$ and for some $x'' \in (x, T)$ we have $v(x'') \leq 0$, hence

$$v(x) > \frac{f(x)}{f(x) + r(1 - F(x))} = \frac{h(x)}{h(x) + r} = \tilde{h}(x) \geq \tilde{h}(x'') \geq v(x''). \tag{35}$$

By the Intermediate-value Theorem, there exists at least one $x^* \in [x, x'']$ with $v(x^*) = \tilde{h}(x^*)$. Denote the smallest one by

$$x^{**} = \min \left\{ x^* \in [x, x''] : v(x^*) = \tilde{h}(x^*) \right\}.$$

By the first inequality in (35), every $x^* \in [x, x^{**}]$ satisfies $v(x^*) \geq \tilde{h}(x^*)$, hence (33) also yields $v'(x^*) \geq 0$, and hence

$$v(x^{**}) \geq v(x) > \tilde{h}(x) \geq \tilde{h}(x^{**}),$$

where the strict inequality is a contradiction.

Step 3. Elimination of Undesired Scenario

By contradiction, suppose that v is strictly increasing on $[x'', T)$ for some $x'' \in (x', T)$. This means, again in view of (33), that $v'(x'') \geq 0$, hence

$$\lim_{x \to T^-} v(x) > v(x'') \geq \tilde{h}(x'') \geq \lim_{x \to T^-} \tilde{h}(x),$$

contradicting (34).

B. Ambiguous Effect of Trading Frequency on Price

Consider the finite-lifetime case $T < \infty$ and suppose that the perceived remaining time W is finite-valued with the distribution F and the density f defined by

$$F(w) = 1 - e^{-\frac{1}{3}\left(w - \frac{T}{2}\right)^3 - \frac{T^3}{24}} \text{ and } f(w) = \left(w - \frac{T}{2}\right)^2 e^{-\frac{1}{3}\left(w - \frac{T}{2}\right)^3 - \frac{T^3}{24}}$$

if $w \in [0, \infty)$. We will show that the equilibrium price p^*_Δ as a function of trading frequency in the sense of Proposition 3 is not monotone.

It suffices to show that an increase in trading frequency can strictly decrease the equilibrium price, because we can reuse the proof of Proposition 3 and see that in the continuous-trade limit the price is the highest by (19) and (12), which specializes to

$$p_\Delta^* = \max_{\substack{x\in[0,T-\Delta] \\ y\in\{\Delta,2\Delta,3\Delta,\ldots,\infty\}\cap[\Delta,T-x]}} g\left(x,y\right). \tag{36}$$

This also reduces the exercise to showing that a decrease in the trade-frequency parameter Δ can make this maximum strictly smaller.

It is enough to find for the trade-frequency parameter a nonempty subinterval of $(T/2\,, T)$ on which the value function of the maximization problem in (36) has a strictly positive derivative (with respect to that parameter). We chose the interval so that (36) simplifies to

$$p_\Delta^* = \max_{x\in[0,T-\Delta]} g\left(x,\Delta\right), \tag{37}$$

for all $\Delta \in (T/2\,, T)$. It is convenient to change a variable and to consider in place of the maximand g the function k on

$$\left\{(x,x') \in \mathbb{R}^2 : 0 \leq x < x' \leq T\right\} \tag{38}$$

into $\mathbb{R}$ defined by

$$k\left(x,x'\right) = g\left(x,x'-x\right).$$

The advantage of expressing the maximand g in terms of k is that it has nice partial derivatives on its domain (38) with the following formulas:

$$k\left(x,x'\right) = \frac{\int_x^{x'} e^{-r(w-x)}dF\left(w\right)}{1-F\left(x\right)-e^{-r(x'-x)}\left(1-F\left(x'\right)\right)},$$

$$\partial_1 k(x,x') = \frac{(f(x) + r(1-F(x)))\,k(x,x') - f(x)}{1 - F(x) - e^{-r(x'-x)}(1-F(x'))}, \tag{39}$$

and

$$\partial_2 k(x,x') = e^{-r(x'-x)} \frac{f(x') - (f(x') + r(1-F(x')))\,k(x,x')}{1 - F(x) - e^{-r(x'-x)}(1-F(x'))}. \tag{40}$$

In turn, the Chain Rule yields nice formulas for the partial derivatives of the maximand g on the interior (as a subset of $\mathbb{R}^2$) of its domain (14):

$$\partial_2 g(x,y) = \partial_2 k(x, x+y),$$

$$\begin{aligned}\partial_1 g(x,y) &= \partial_1 k(x,x+y) + \partial_2 k(x,x+y) \\ &= rg(x,y) + \frac{e^{-ry} f(x+y) - f(x)}{1 - F(x) - e^{-ry}(1-F(x+y))}(1 - g(x,y)),\end{aligned}$$

and

$$\begin{aligned}\partial_{11} g(x,y) = &\left(r - \frac{e^{-ry} f(x+y) - f(x)}{1 - F(x) - e^{-ry}(1-F(x+y))}\right)\partial_1 g(x,y) + (1-g(x,y)) \\ &\times\left(\frac{e^{-ry} f'(x+y) - f'(x)}{1 - F(x) - e^{-ry}(1-F(x+y))} - \left(\frac{e^{-ry} f(x+y) - f(x)}{1 - F(x) - e^{-ry}(1-F(x+y))}\right)^2\right).\end{aligned}$$

We also need them at the boundary point $(0,T)$:

$$\partial_2 g(0,T) = \lim_{\substack{x\to 0^+ \\ y \to T^-}} \partial_2 g(x,y) = e^{-rT} \frac{f(T) - (f(T) + r(1-F(T)))\,g(0,T)}{1 - e^{-rT}(1-F(T))}, \tag{41}$$

$$\partial_1 g(0,T) = \lim_{\substack{x\to 0^+ \\ y \to T^-}} \partial_1 g(x,y) = rg(0,T) + \frac{e^{-rT} f(T) - f(0)}{1 - e^{-rT}(1-F(T))}(1 - g(0,T)),$$

and

$$\begin{aligned}
\partial_{11} g(0,T) &= \lim_{\substack{x\to 0^+\\ y\to T^-}} \partial_{11} g(x,y) \\
&= \left(r - \frac{e^{-rT} f(T) - f(0)}{1-e^{-rT}(1-F(T))}\right) \partial_1 g(0,T) \\
&\quad + \left(\frac{e^{-rT} f'(T) - f'_+(0)}{1-e^{-rT}(1-F(T))} - \left(\frac{e^{-rT} f(T) - f(0)}{1-e^{-rT}(1-F(T))}\right)^2\right)(1-g(0,T)) \\
&= \left(r + \frac{T^2}{4}\right) \partial_1 g(0,T) + \frac{1+e^{-rT-\frac{T^3}{12}}}{1-e^{-rT-\frac{T^3}{12}}} T(1-g(0,T)).
\end{aligned}$$

Two scenarios on their signs are possible: either $\partial_1 g(0,T) < 0$ or else $\partial_{11} g(0,T) > 0$, because

$$g(0,T) = \frac{\int_0^T e^{-rw} dF(w)}{1-e^{-rT}(1-F(T))} < 1.$$

The implication for the maximization problem in (37) is that there exists a $\delta \in (0,\infty)$ such that any $\Delta \in (T/2\,, T) \cap (T-\delta, T)$ makes the maximand $g(\cdot,\Delta)$ on $[0, T-\Delta]$ either strictly decreasing or else strictly convex. In either scenario, interior maximizers are absent, but there are two cases to consider according as 0 is a maximizer for all sufficiently large $\Delta \in (0,T)$ or not (Cases 1 and 2 respectively):

Case 1 ($p^*_\Delta = g(0,\Delta)$ eventually as $\Delta \to T^-$). It suffices to show, in view of (41), that $\partial_2 g(0,T) > 0$, or, equivalently, that

$$g(0,T) < \frac{f(T)}{f(T) + r(1-F(T))}. \tag{42}$$

We save the proof, which overlaps with (the opposite) Case 2, for the end of this exercise.

Case 2 ($p^*_\Delta \neq g(0,\Delta)$ frequently as $\Delta \to T^-$). It means that for every $\varepsilon \in (0,\delta]$ there exists a $\Delta \in (T/2\,, T) \cap (T-\varepsilon, T)$ such that $p^*_\Delta \neq g(0,\Delta)$. The only possibility left is that for every $\varepsilon \in (0, \delta]$ there exists a $\Delta \in (T/2\,, T) \cap (T - \varepsilon, T)$ such that $p^*_\Delta = g(T - \Delta, \Delta) > g(0, \Delta)$. In fact, this inequality ensures that for every $\varepsilon \in (0, \delta]$ there exist $y, y' \in (T/2\,, T) \cap (T - \varepsilon, T)$ with $y < y'$ such that all $\Delta \in [y, y']$ satisfy

$$p^*_\Delta = g(T - \Delta, \Delta) = k(T - \Delta, T)\,.$$

Since $\varepsilon \in (0, \delta]$ was arbitrary, it suffices to show, in view of (39), that $\partial_1 k(0, T) < 0$, or, equivalently, that

$$k(0, T) < \frac{f(0)}{f(0) + r(1 - F(0))}.$$

It turns out simply by definitions that this inequality and (42) in Case 1 are equivalent, and thus it suffices to prove (42), which goes by showing that the reverse inequality

$$g(0, T) \geq \frac{f(T)}{f(T) + r(1 - F(T))}, \tag{43}$$

or, equivalently, $\partial_2 g(0, T) \leq 0$, is a contradiction. First of all, the right-hand side of the former inequality equals $g(T, 0)$, as all points $x \in [0, T]$ satisfy

$$g(x, 0) = \frac{f(x)}{f(x) + r(1 - F(x))} = \frac{\left(x - \frac{T}{2}\right)^2}{\left(x - \frac{T}{2}\right)^2 + r}, \tag{44}$$

hence it also follows that $\partial_2 g(0, T) - \partial_1 g(T, 0) < 0$. These observations together with inequality (43) yields a $\delta \in (0, \infty)$ such that all $x' \in [0, T] \cap (T - \delta, T)$ satisfy

$$g(0, x') > \frac{f(x')}{f(x') + r(1 - F(x'))}. \tag{45}$$

As a next step, this inequality extends to all $x' \in (0, T)$, since otherwise there are

$x' \in (0, T)$ with

$$g(0, x') \leq \frac{f(x')}{f(x') + r(1 - F(x'))},$$

they have a maximum $x \leq T - \delta$, all $x' \in (x, T)$ satisfy the reverse inequality (45), in view of (40) we have $\partial_2 k(0, x') < 0$, hence in view of (43) and (44) have the absurd

$$g(0, x) = k(0, x) > k(0, T) = g(0, T) \geq g(T, 0) > g(x, 0) = \frac{f(x)}{f(x) + r(1 - F(x))}.$$

All $x' \in (0, T)$ must satisfy (45) and, as in the last step, $\partial_2 k(0, x') < 0$, which, together with (44), implies that

$$g(T, 0) = g(0, 0) > g(0, T),$$

contradicting the inequality in (43) and completing the calculations for this example.

References

Aliprantis, C. D. and K. C. Border (2006). *Infinite Dimensional Analysis: A Hitchhiker's Guide*. Springer.

Allen, F., S. Morris, and A. Postlewaite (1993). "Finite Bubbles with Short Sale Constraints and Asymmetric Information". *Journal of Economic Theory* 61(2), pp. 206–229.

Allen, F. and G. Gorton (1993). "Churning Bubbles". *Review of Economic Studies* 60(4), pp. 813–836.

Barlevy, G. (2015). "Bubbles and Fools". *Economic Perspectives* 39(2), pp. 54–76.

Berestycki, H., C. Bruggeman, R. Monneau, and J. A. Scheinkman (2019). "Bubbles in Assets with Finite Life". *Mathematics and Financial Economics* 13(3), pp. 429–458.

Billingsley, P. (1995). *Probability and Measure*. A Wiley-Interscience Publication. Wiley.

Chung, K. L. (2001). *A Course in Probability Theory*. Academic Press.

Collin-Dufresne, P., M. Johannes, and L. A. Lochstoer (2016). "Asset Pricing When 'This Time Is Different'". *Review of Financial Studies* 30(2), pp. 505–535.

Ehling, P., A. Graniero, and C. Heyerdahl-Larsen (Dec. 2017). "Asset Prices and Portfolio Choice with Learning from Experience". *Review of Economic Studies* 85(3), pp. 1752–1780.

Harrison, J. M. and D. M. Kreps (1978). "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations". *Quarterly Journal of Economics* 92(2), pp. 323–336.

Hong, H., J. A. Scheinkman, and W. Xiong (2006). "Asset Float and Speculative Bubbles". *Journal of Finance* 61(3), pp. 1073–1117.

Jarvis, P. (Jan. 26, 2021). "GameStop's 400% Rally Puts Market Value atop Some in S&P 500". *BNN Bloomberg*. URL: `https://www.bnnbloomberg.ca/gamestop-s-400-rally-puts-market-value-atop-some-in-s-p-500-1.1554001` (visited on 10/22/2021).

Liu, F. and J. R. Conlon (2018). "The Simplest Rational Greater-fool Bubble Model". *Journal of Economic Theory* 175, pp. 38–57.

Malmendier, U. (2021). "Experience Effects in Finance: Foundations, Applications, and Future Directions". *Review of Finance* 25(5), pp. 1339–1363.

Malmendier, U., D. Pouzo, and V. Vanasco (2020). "Investor Experiences and Financial Market Dynamics". *Journal of Financial Economics* 136(3), pp. 597–622.

Mas-Colell, A., M. D. Whinston, and J. R. Green (1995). *Microeconomic Theory*. Oxford University Press.

Michaely, R. and A. Moin (2021). "Disappearing and Reappearing Dividends". *Journal of Financial Economics*.

Moore, J. (2010). *Mathematical Methods for Economic Theory 2*. Studies in Economic Theory. Springer.

Morris, S. (1992). "The re-sale premium for assets in general equilibrium". Unpublished.

Morris, S. (1995). "The Common Prior Assumption in Economic Theory". *Economics and Philosophy* 11(2), pp. 227–253.

Morris, S. (1996). "Speculative Investor Behavior and Learning". *Quarterly Journal of Economics* 111(4), pp. 1111–1133.

Nakov, A. and G. Nuño (2015). "Learning From Experience in the Stock Market". *Journal of Economic Dynamics and Control* 52, pp. 224–239.

Newburger, E. (June 4, 2019). "GameStop Stock Plunges Nearly 30% as Sales Fall Short of Estimates and Its Dividend Is Eliminated". *CNBC*. URL: `https://www.cnbc.com/2019/06/04/gamestop-stock-falls-as-sales-fall-short-and-dividend-eliminated.html` (visited on 10/26/2021).

Nutz, M. and J. A. Scheinkman (2020). "Shorting in Speculative Markets". *Journal of Finance* 75(2), pp. 995–1036.

Oehmke, M. and A. Zawadowski (2017). "The Anatomy of the CDS Market". *Review of Financial Studies* 30(1), pp. 80–119.

Samuelson, P. A. (1958). "An Exact Consumption-loan Model of Interest with or without the Social Contrivance of Money". *Journal of Political Economy* 66(6), pp. 467–482.

Santos, M. S. and M. Woodford (1997). "Rational Asset Pricing Bubbles". *Econometrica* 65(1), pp. 19–57.

Scheinkman, J. A. and W. Xiong (2003). "Overconfidence and Speculative Bubbles". *Journal of Political Economy* 111(6), pp. 1183–1220.

Schraeder, S. (2016). "Information Processing and Non-Bayesian Learning in Financial Markets". *Review of Finance* 20(2), pp. 823–853.

Steiner, J. and C. Stewart (2015). "Price Distortions under Coarse Reasoning with Frequent Trade". *Journal of Economic Theory* 159, pp. 574–595.

Tirole, J. (1985). "Asset Bubbles and Overlapping Generations". *Econometrica* 53(6), pp. 1499–1528.

Tversky, A. and D. Kahneman (1974). "Judgment under Uncertainty: Heuristics and Biases". *Science* 185(4157), pp. 1124–1131.

Werner, J. (2020). "Speculative Bubbles, Heterogeneous Beliefs, and Learning". Unpublished.

Xiong, W. and J. Yu (2011). "The Chinese Warrants Bubble". *American Economic Review* 101(6), pp. 2723–2753.