Dr. Sebastian Schlenkrich
Institut für Mathematik
Bereich Stochastik

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Interest Rate Modelling and Derivative Pricing, WS 2018/19

Exercise 6

1. **Bermudan options**
Consider the pricing of a Bermudan option with n exercise times $T_1, \ldots, T_n$. Let $k \in \{0, \ldots. n\}$ and set $T_0 = 0$. Denote $U_k(T_k)$ the time-T_k value of the underlying which the option holder receives if she exercises the option at T_k. Further denote $H_k(T_k)$ the time-T_k value of the option where the exercise times are restricted to $T_{k+1}, \ldots T_N$ ($H_k(T_k)$ is the option continuation value). We set $H_k(t) = B(t) \cdot \mathbb{E}_t [H_k(T_k)/B(T_k)]$ for $t \leq T_k$ and expectation $\mathbb{E}_t [\cdot]$ conditional on time-t. The Bermudan option price is derived via the recursion

$$H_k(T_k) = B(T_k) \cdot \mathbb{E}_{T_k} \left[\frac{H_k(T_{k+1})}{B(T_{k+1})} \right] = B(T_k) \cdot \mathbb{E}_{T_k} \left[\frac{\max\{U_{k+1}, H_{k+1}(T_{k+1})\}}{B(T_{k+1})} \right]$$

for $k = N-1, \ldots 0$.

Show that the Bermudan option price $H_0(0)$ can be written as

$$H_0(0) = B(0) \cdot \sum_{k=1}^{n} \mathbb{E} \left[\frac{[U_k(T_k) - H_k(T_k)]^+}{B(T_k)} \right]$$

2. **Accuracy of Density Integration Methods**
Consider the pricing of a $10y$ expiry into $10y$ maturity coupon bond call option (i.e. bond payments in $11y$, $12y$, ...,$20y$). Chose interest rates such that the (forward) bond is at-the-money and calculate the option price in a Hull White model via analytic formula. Also set up the coupon bond option as Bermudan option (with a couple of trivial additional exercises) and use density integration methods to calculate the option price.

   (a) Compare the pricing accuracy of Simpson's method (with and without break-even point calculation) for various choices of number of grid points and grid sizes.

   (b) Compare the pricing accuracy of Hermite integration method for various choices of number of grid points and Hermite polynomial degrees.

   (c) Compare the pricing accuracy of exact cubic spline integration method (with and without break-even point calculation) for various choices of number of grid points and grid sizes.

   (d) Which integration method would you recommend in terms of accuracy and computational effort?

3. **Accuracy of PDE Methods**
Consider the pricing of a $10y$ expiry into $10y$ maturity coupon bond call option (i.e. bond payments in $11y$, $12y$, ...,$20y$). Chose interest rates such that the (forward) bond is at-the-money and calculate the option price in a Hull White model via analytic formula. Also set up the coupon bond option as Bermudan option (with a couple of trivial additional exercises) and use PDE methods to calculate the option price.

   (a) Compare the pricing accuracy of PDE method for various choices of number of grid points, grid sizes and time step sizes.

(b) How should the time step size be chosen in relation to the number of grid points and/or grid size?

(c) How does the choice of θ in the time integration impact the accuracy?

(d) Analyse the impact of the choice of boundary condition ($\lambda_{0,N} = 0$ or via bond-option-approximation) versus the grid size (in terms of standard deviations) on the accuracy of the numerical method.

4. **Bermudan Swaption Pricing**
Analyse and explain the pricing of Bermudan swaptions implemented in `BermudanSwaption.py` (assume a model is provided, i.e. without model calibration). Use the Python code in `testBermudanSwaption.py` to setup in-the-money, at-the-money and out-of-the-money swaptions. Compare the Bermudan price relative to the co-terminal European bond option prices.