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Exercise 7

1. **Convergence of Monte-Carlo Method**
   Consider the pricing of a $10y$ expiry into $10y$ maturity coupon bond call option (i.e. bond payments in $11y$, $12y$, ...,$20y$). Chose interest rates such that the (forward) bond is at-the-money and calculate the option price in a Hull White model via analytic formula. Also set up the coupon bond option as Bermudan option with a single exercise and use Monte-Carlo method to calculate the option price.

   (a) Analyse the convergence of the Monte-Carlo method to the analytical option price for increasing number of paths.

   (b) How does the convergence change if you change the Hull White model volatility and mean reversion?

2. **Accuracy of American Monte-Carlo Methods**
   Consider the pricing of a $10y$ expiry into $10y$ maturity Bermudan coupon bond call option with annual call right. (i.e. bond payments in $11y$, $12y$, ...,$20y$, exercises in $10y$, $11y$, ...,$19y$). Choose interest rates such that the (forward) bond is at-the-money. Calculate a reference option price in a Hull White model via PDE (or density integration) method with high accuracy. Also calculate the Bermudan option price using American Monte-Carlo method.

   (a) Analyse the convergence of the Monte-Carlo method to the reference option price for increasing number of paths.

   (b) How does the price change if you change the max. polynomial order for regression?

   (c) How does the price change depending on regression on the continuation value compared to regression on the exercise decision only?

   (d) Add a modified American Monte-Carlo method that uses the following basis functions:

      i. a swap rate $S(T_E)$ from exercise to final maturity (i.e. co-terminal swap rate), and

      ii. the term $[S(T_E) - K]^+$, where K is the fixed rate of the underlying coupon bond.

   Analyse the impact on the pricing for increasing max. polynomial order for regression. How does the pricing conpare to the state variable approach?