1. Proof Theorem 2.2.24 from the lecture notes.

2. Let \( W \) be a three-dimensional Brownian motion starting in \( x \neq 0 \). Let \( u : \mathbb{R}^3 \setminus \{0\} \to \mathbb{R} \) be defined by \( u(x) := \|x\|^{-1} \). Prove that \( X_t := u(W_t) \) defines a local martingale that is not a true martingale.

   \[ \text{Hint: Use the fact that if a random variable } Z \text{ is } \chi^2 \text{ distributed with } t \text{ degrees of freedom, then } Y := \frac{1}{Z} \text{ is inverse } \chi^2 \text{ distributed with } t \text{ degrees of freedom } (t > 2), \text{ and } E[Y] = \frac{1}{t-2}. \text{ In particular, } u(W_t) \text{ is inversely } \chi^2 \text{ distributed with } 3t - 2 \text{ degrees of freedom } (3t \geq 2). \]

3. Let \( f \in \mathcal{V} \) be bounded and let \( M_t := \int_0^t f(s, \omega) dW_s \). Prove “by hand” that

\[
M_t^2 - \int_0^t v^2(s, \omega) ds
\]

is a martingale.