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STOCHASTISCHE FINANZMATHEMATIK II Sommersemester 2019

ÜBUNGSBLATT 1

1. Proof Theorem 2.2.24 from the lecture notes.

2. Let W be a three-dimensional Brownian motion starting in $x \neq 0$. Let $u : \mathbb{R}^3 \backslash \{0\} \to \mathbb{R}$ be defined by $u(x) := \|x\|^{-1}$. Prove that $X_t := u(W_t)$ defines a local martingale that is not a true martingale.

   Hint: Use the fact that if a random variable Z is χ^2 distributed with t degrees of freedom, then $Y := \frac{1}{Z}$ is inverse χ^2 distributed with t degrees of freedom ($t > 2$), and $E[Y] = \frac{1}{t-2}$. In particular, $u(W_t)$ is inversely χ^2 distributed with $3t - 2$ degrees of freedom ($3t \geq 2$).

3. Let $f \in \mathcal{V}$ be bounded and let $M_t := \int_0^t f(s, \omega) dW_s$. Prove "by hand" that

$$M_t^2 - \int_0^t v^2(s, \omega) ds$$

   is a martingale.