# Course: Branching processes

**Instructor:**
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## Content

This course mainly gives a detailed introduction about branching processes, which are an important class of Markov processes originating from the stochastic modeling of population dynamics and have been widely applied in the study of random network, queue theory and finance. This course starts from introducing the discrete-state Galton-Watson branching processes (GW-processes) and their elementary properties such as criticality, extinction probability and (conditional-) limit theorems. Then we construct continuous-state branching processes (CB-processes) as scaling limits of GW-processes. We will also introduce some elementary properties of CB-processes, such as extinction probabilities, limit theorems, extreme distributions and criterion for transience or recurrence. After that we reconstruct CB-processes in another three different ways: excursion reconstruction, semigroups representation and stochastic equation representation. Then we introduce a fundamental result: Lamperti's transformations by time changes, which assert that CB-processes are in one-to-one correspondence with spectrally positive Lévy processes via simple random time changes. At the end of this course, we will introduce some applications of branching processes including Ray-Knight theorem for random walks and Brownian motion, term structure of interest rate, cluster representations for Hawkes processes and branching representations for M/G/n-systems.

## Prerequisites

- Either Stochastics I and II or Stochastic Analysis is necessary
- Undergraduate Real Analysis and Functional Analysis are assumed
- Some familiarity with ordinary differential equations would be helpful

# References

Lecture notes will be available as the course progresses. Other good books can be referred.

1. Athreya, K.B. and Ney, P.E. (1972): *Branching Processes.* Springer, Berlin
2. Bansaye, V. and Méléard, S. (2015): *Stochastic Models for Structured Populations: Scaling Limits and Long Time Behavior.* Springer.
3. Li, Z. (2011): *Measure-Valued Branching Markov Processes.* Springer, Heidelberg.
4. Pardoux, E. (2015): *Probabilistic Models of Population Evolution: Scaling Limits, Genealogies and Interactions.* Springer.