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Interest Rate Modelling and Derivative Pricing, WS 2020/21

Exercise 2

1. **Schedules and fixed leg pricing**
   Consider the following data of a fixed rate cash flow leg:

Start	43038	**DCF**
Pay/Start	43403	146,810.49
Pay/Start	43768	143,185.72
Pay/Start	44134	140,023.47
Pay/Start	44498	135,829.32
Pay/Start	44862	132,484.75
Pay/Start	45229	130,260.80
Pay/Start	45595	126,689.80
Pay/Start	45960	123,224.21
Pay/Start	46325	120,181.80
Pay/Start	46689	116,901.37
Pay/Start	47056	114,939.00
Pay/Start	47421	111,490.24
Pay/Start	47786	108,737.54
Pay/Start	48151	106,052.80
Pay/Start	48516	103,434.35
Pay/Start	48883	101,419.42

   Here 'Pay/Start' represents the payment and coupon period start dates of the coupon schedule derived based on the TARGET holiday calendar. The first date represents the first coupon start date. The dates are given as serial numbers with '1' representing January 1, 1900.
   The column 'DCF' contains discounted cash flow values that are derived via

$$DCF = N \times r \times \tau(d_{Start}, d_{End}) \times DF(d_{End}).$$

   Here N is the leg notional, r is the fixed interest rate, $\tau(d_{Start}, d_{End})$ is the year fraction between start and end date and $DF(d_{End})$ is the discount factor to payment date.

   (a) Determine the properties of the schedule that generates above schedule dates:
   - What is the schedule frequency?
   - What is the business day convention?
   - What is the generation rule? Are there any (long/short) front or back stubs?

   Hint: Use the `QuantLib.schedule(...)` routine. For details on the interface, see `https://github.com/lballabio/QuantLib-SWIG/blob/master/SWIG/scheduler.i`

   (b) Assume valuation date is October 2, 2017, $N = 3.500.000$ and $r = 4.25\%$. Moreover, assume a yield curve parametrised by a flat forward rate (continuous compounding, Act/365 (Fixed) day count convention). What is the day count convention and flat forward rate that generates above discounted cash flows?

2. **Float leg pricing**
   Consider the setup in `testSwapPricing.py`. Set up several example Vanilla swaps and verify numerically the relation that - in a single curve yield curve setting - the float leg NPV can be written as
   $$V^{\text{FloatLeg}}(t) = \sum_{i=1}^{N} P(t, T_i) \cdot L(t, T_{i-1}, T_i) \cdot \tau_i = P(t, T_0) - P(t, T_N)$$
   where $t = 0$ is the valuation time, T_0 the float leg start time and T_N the float leg end time.
   Vary the float leg business day and day count convention. What is the impact on the validity of above float leg pricing relation?
   Hint: You can either use the cash flow details provided by `Swap.floatCashFlows()` and discount cash flows manually or you can use the the routine `ql.VanillaSwap.floatingLegNPV()` to get the float leg NPV directly.

3. **Multi-curve Libor rate modelling**
   Consider the Libor rate
   $$L^{\delta}(T; T_0, T_1) = \left[\frac{P(T, T_0)}{P(T, T_1)} \cdot D(T_0, T_1) - 1 \right] \frac{1}{\tau}$$
   that takes into account credit (or alternatively funding) risk via the deterministic basis term $D(T_1, T_2)$.

   (a) Determine the pricing measure under which $L^{\delta}(T; T_0, T_1)$ is a martingale, show the martingale property and derive the formula of the time-t forward Libor rate.

   (b) Alternatively, tenor-dependent Libor rates are sometimes modelled via
   $$\hat{L}^{\delta}(T; T_0, T_0 + \delta) = L(T; T_0, T_0 + \delta) + s^{\delta}(T_0).$$
   Here $s^{\delta}(T_0)$ is a deterministic spread over the credit-risk-free Libor rate $L(T; T_0, T_1) = [P(T, T_0)/P(T, T_1) - 1] / \tau$. Show that $\hat{L}^{\delta}(T; T_0, T_1)$ is a martingale in the same martingale measure as in (a).

   (c) Suppose $L^{\delta}(T; T_0, T_1)$ and $\hat{L}^{\delta}(T; T_0, T_0 + \delta)$ model the same Libor rate. How is the multiplicative basis spread $D(T_0, T_1)$ related to the additive basis spread $s^{\delta}(T_0)$? Can we specify a model in which Libor rates are stochastic and both spreads $D(T_0, T_1)$ and $s^{\delta}(T_0)$ are deterministic?

4. **Impact of interpolation on float leg and fixed leg pricing**
   Consider the alternative yield curve interpolation methods from Exercise 1.1 and the swap pricing example in `testSwapPricing.py`.

   (a) Analyse the impact of the choice of interpolation method on the fixed leg and floating leg cash flows.

   (b) How do these observations relate to your observations from Exercise 1.1? Do these observations change your reasoning about a preferred interpolation method? Explain your conclusions.