New Conditions for the Existence of Radner Equilibrium with Infinitely Many States

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Abstract

In atomless differential information economies, equilibria are known not to exist prevalently even when agents are risk averse expected utility maximizers. The notion of prevalence involves essentially picking an economy at random. In this paper, however, we establish existence results with economically meaningful assumptions on the information structure. We obtain existence when agents have independent information, and also when the total endowment of the economy is common knowledge.

Keywords: differential information; asymmetric information; independent information; common knowledge; no-trade theorems; competitive equilibrium

1 Introduction

In 1968 *Radner* explored how far one could go in applying the theory of competitive equilibrium to the case of differentially informed agents. He tailored a way to model information asymmetries so that the standard notion of Walrasian equilibrium would apply. His economic model has subsequently become known as a differential information economy, and its Walrasian equilibrium is commonly referred to as Radner equilibrium (see Section 2). *Radner* concluded that standard theorems on the existence of Walrasian equilibrium continued to hold, but that referred only to the case of finitely many states of nature and finitely many state-independent commodities (available for consumption in each state), as infinite-dimensional equilibrium theory was only in its infancy back then.

It is now understood that the situation with infinite-dimensional commodity spaces is more subtle. *Podczeck and Yannelis* (2008) established the existence of Radner equilibrium with infinitely many state-independent commodities, but

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the case of infinitely many states of nature was left behind. *Tourky and Yannelis* (2003) and *Podczeck et al.* (2008) have discovered prevalent non-existence conditions peculiar to the case of infinitely many states even with preferences confined to risk averse expected utility[1]. They utilize the notion of prevalence introduced by *Anderson and Zame* (2001). The basic idea behind this result is that in atomless differential information economies agents with different priors and information seek to specialize their optimal consumption on a null event, and the duality condition characterizing the existence of equilibrium derived in *Aliprantis et al.* (2004, 2005) cannot hold.

Radner equilibrium with infinitely many states is nevertheless known to exist under restrictive yet economically meaningful assumptions on the information structure. Such an assumption was found by *Hervés-Beloso et al.* (2009). They suppose that each agent observes a public and a private signal; the public signal may take infinitely many values, but private signals are restricted to take only finitely many values. We introduce two new economically meaningful conditions that guarantee the existence of Radner equilibrium with infinitely many states.

Our first condition (Section 3) requires that agents' information σ-algebras (or signals) are independent. If, in addition, there is only one commodity available for consumption in each state, then there exists a unique Radner equilibrium in which there is no trade. With more commodities per state, however, agents might be willing to trade, and the problem of existence becomes more challenging. In this more general scenario, we also make an assumption that limits the substitutability of one state-independent commodity by others.

Our second condition (Section 4) requires that the total endowment of the economy is common knowledge. This is the same as saying that the total endowment belongs to every agent's informationally constrained consumption set. We also make somewhat unusual assumptions on preferences, but we show that they are implied by standard assumptions if agents exhibit a degree of risk aversion. In particular, this risk aversion is satisfied in the standard case of expected utility with concave Bernoulli functions. We also give an example of non-expected utility preferences satisfying all our assumptions.

Let us briefly mention the importance of infinite state spaces. They arise naturally, for instance, if uncertainty is resolved sequentially over an infinite horizon (see *Shreve*, 2004). In addition, they are often utilized for the sake of mathematical convenience. For example, if one wants to work with continuously distributed random variables (say, agents' signals about the return of a risky asset), then the state space must be infinite. These considerations originally motivated *Bewley* (1972) and other authors to introduce infinite-dimensional commodity spaces into general equilibrium theory.

The existence of Radner equilibrium is an active area of research. Recent contributions include *Xanthos* (2014) and *Yoo* (2013).

[1]The author thanks professor Rabee Tourky for kindly providing these working papers.

2 Model

We have a finite set of *agents* $I = \{1, \dots, m\}$ who face exogenous *uncertainty* described by a probability space $(S, \mathscr{F}, \mu)$. There is a finite nonempty set C of *state-independent commodities* available for consumption in each state. Agents are differentially informed, which simply means that they have heterogeneous ability to discern events in $\mathscr{F}$. Agent i can only discern events that belong to some sub-σ-algebra $\mathscr{F}_i$ of $\mathscr{F}$. If $\mathscr{G}$ is any sub-σ-algebra of $\mathscr{F}$, we denote the space $(L_1(S, \mathscr{G}, \mu|_{\mathscr{G}}))^C$ by $L_1(\mathscr{G})$ and its positive cone by $L_1^+(\mathscr{G})$. Notice that we can identify points $x \in L_1(\mathscr{G})$ and $y \in L_1(S, \mathscr{G}, \mu|_{\mathscr{G}}, \mathbb{R}^C)$ satisfying $x_c(s) = (y(s))_c$ for all $s \in S$ and $c \in C$. We take $L_1(\mathscr{F})$ as our *commodity space* and let $L_1^+(\mathscr{F}_i)$ be the informationally constrained *consumption set* of agent i. The agent has a *preference correspondence* $P_i : L_1^+(\mathscr{F}_i) \twoheadrightarrow L_1^+(\mathscr{F}_i)$ and an *initial endowment* $\omega_i \in L_1^+(\mathscr{F}_i)$. We let $\omega = \sum_{i=1}^m \omega_i$ denote the total initial endowment.

An *allocation* is a vector $x \in \Pi_{i=1}^m L_1^+(\mathscr{F}_i)$ such that $\sum_{i=1}^m x_i = \omega$. A *price system* is an element of $(L_\infty(S, \mathscr{F}, \mu))^C$, the topological dual of $L_1(\mathscr{F})$. Given a price system p, the value of a commodity bundle $x \in L_1(\mathscr{F})$ is simply $p \cdot x = \sum_{c \in C} E(p_c x_c)$. An allocation x is said to be a *Radner equilibrium* if there exists a price system p such that, for all $i \in I$, we have $p \cdot x_i \leq p \cdot \omega_i$, and $y \in P_i(x_i)$ implies $p \cdot y > p \cdot \omega_i$.

A remark about the interpretation of the probability measure μ is in order now. Technically, our model would not change if we replaced μ by another measure ν as soon as μ-null sets coincided with ν-null sets. This is because spaces of (equivalence classes of) μ-integrable and ν-integrable random variables are lattice isometric. But concepts of independence and risk aversion, which are of fundamental importance to the theory of choice under uncertainty and are in use in this paper, are not immune to such a change of measures. If two random variable are independent with respect to μ, they are not necessarily independent with respect to ν. If a preference relation is risk averse with respect to μ, it is not necessarily risk averse with respect to ν. To be able to interpret mathematical independence as a reflection of causal independence in the real world, we must suppose that μ is a "true" probability measure. On the other hand, the assumption of risk aversion with respect to μ is hard to justify unless μ is supposed to be our agents' common belief. So we can view μ as a "true" probability measure in Section 3, where we make use of the independence assumption, and as our agents' common belief in Section 4, where risk aversion comes into play.

We use expected values and conditional expectations extensively in the exposition of our results. If $x \in L_1(\mathscr{F})$ and $\mathscr{G}$ is a sub-σ-algebra of $\mathscr{F}$, the symbol $E(x)$ denotes the vector $a \in \mathbb{R}^C$ in which $a_c = E(x_c)$ for all $c \in C$, while the notation $E(x|\mathscr{G})$ stands for the element $y \in L_1(\mathscr{G})$ in which $y_c = E(x_c|\mathscr{G})$ for all $c \in C$.

We conclude this section by listing below standard assumptions that would typically be required for an existence proof even without information asymmetries, i.e. when $\mathscr{F}_i = \mathscr{F}_j$ for all $i, j \in I$, as in Section 9.1 of *Aliprantis et al.*

(2001). Assumption (A6) is a version of the properness condition introduced by *Tourky* (1998). We will refer to these assumptions later.

(A) The following is true for every $i \in I$ and some $v \in \Pi_{j=1}^{m} L_1(\mathscr{F}_j)$ satisfying $\sum_{j=1}^{m} v_j \leq \omega$ and $v_j > 0$ for all $j \in I$.

  (1) P_i is *irreflexive*, i.e. $x \notin P_i(x)$ for all $x \in L_1^+(\mathscr{F}_i)$.

  (2) P_i is *convex-valued*, i.e. $P_i(x)$ is a convex set for all $x \in L_1^+(\mathscr{F}_i)$.

  (3) P_i is *strictly monotone*, i.e. $x \in L_1^+(\mathscr{F}_i)$ implies $x + y \in P_i(x)$ for all $y \in L_1^+(\mathscr{F}_i) \setminus \{0\}$.

  (4) P_i has *open values*, i.e. $P_i(x)$ is open in $L_1^+(\mathscr{F}_i)$, relative to a linear topology on $L_1(\mathscr{F})$, for all $x \in L_1^+(\mathscr{F}_i)$.

  (5) P_i has *weakly open lower sections*, i.e. for every $z \in L_1^+(\mathscr{F}_i)$ the set $P_i^{-1}(z) = \left\{y \in L_1^+(\mathscr{F}_i) : z \in P_i(y)\right\}$ is weakly open in $L_1^+(\mathscr{F}_i)$.

  (6) P_i is *proper* in the sense that there exists a convex-valued correspondence $\hat{P}_i : L_1^+(\mathscr{F}_i) \twoheadrightarrow L_1(\mathscr{F})$ such that for each $x \in L_1^+(\mathscr{F}_i)$

    (i) $x + v_i$ is an interior point of $\hat{P}_i(x)$ and

    (ii) $\hat{P}_i(x) \cap L_1^+(\mathscr{F}_i) = P_i(x)$.

3 Independent Information

In this section we establish the existence of Radner equilibrium when agents' information σ-algebras are independent. We consider the case of a single commodity per state separately first. In this scenario we obtain the existence of a unique Radner equilibrim in which there is no trade. This result is in accord with *Koutsougeras and Yannelis* (1993), who prove that only the initial allocation belongs to the private core when agents have independent information[2]. However, our main contribution is to show that the initial allocation can actually be supported by some price system. We cannot resort to the second welfare theorem or the converse part of the core equivalence theorem to obtain supporting prices. *Tourky and Yannelis* (2003) and *Podczeck et al.* (2008) demonstrate that these theorems do not hold for differential information economies with infinitely many states.

With a single commodity per state, we require only pairwise independence of agents' information. This assumption is expressed formally in (B) below.

Assumption (C), with a single commodity per state and monotonicity (A3), is satisfied whenever in the single-agent economy corresponding to each agent i we can find a Walrasian equilibrium. Technically, this assumption requires the strict separation of the sets $\{\omega_i\}$ and $P_i(\omega_i)$, which need not be convex, by a normalized continuous linear functional. This is implied by a wide variety of conditions on preferences. One set of sufficient conditions for Assumption

[2]The author thanks professor Nicholas Yannelis for bringing this result to his attention.

(C) is given by Assumption (A) with v_i replaced by ω_i (*Aliprantis et al.*, 2001, Corollary 9.2).

Assumption (D) is used in the proof of uniqueness only. This monotonicity condition is implied by strict monotonicity as stated in (A3).

(B) *Pairwise independence:* $\mu(F_i \cap F_j) = \mu(F_i)\,\mu(F_j)$ for any choice of $F_i \in \mathscr{F}_i$ and $F_j \in \mathscr{F}_j$, for all $i, j \in I$ such that $i \neq j$.

(C) *Separation:* there exist price systems p_i, for each $i \in I$, such that $y \in P_i(\omega_i)$ implies $p_i \cdot y > p_i \cdot \omega_i$, and $E(p_i) = E(p_j)$ for all $i, j \in I$.

(D) *Monotonicity:* for every $i \in I$ and for all scalars $\alpha > 0$, if $y \in L_1^+(\mathscr{F}_i)$ is such that $y_c(s) = \omega_{ic}(s) - \alpha$ for almost all $s \in S$ for all $c \in C$, then $\omega_i \in P_i(y)$.

Theorem 1. *Suppose that there is a single commodity per state, i.e. the set C is a singleton. If Assumptions (B) and (C) hold, then the initial allocation $(\omega_1, \ldots, \omega_m)$ is a Radner equilibrium. If, in addition, Assumption (D) holds, then this Radner equilibrium is unique.*

There is no trade because with independent information each agent's net trade must be constant across states (almost everywhere), for otherwise either the agent or the rest of the agents are unable to verify the trade. Since agreeing to a negative net trade contradicts individual rationality (with monotone preferences), and strictly positive net trades are infeasible, net trades must be zero in every individually rational allocation.

When the number of commodities per state is greater than one, then each agent's net trade in each commodity must be still constant across states. However, now a negative net trade in one commodity may be compensated with a positive net trade in another. In other words, the initial allocation need not be a Radner equilibrium. Technically, the normalization requirement in Assumption (C) makes it too strong.

In the case of many commodities per state, we will construct a Radner equilibrium from a personalized equilibrium, supported by a possibly non-linear value function, of *Aliprantis et al.* (2001). Let us define these concept precisely. An *allocation with free disposal* is a vector $x \in \Pi_{i=1}^m L_1^+(\mathscr{F}_i)$ such that $\sum_{i=1}^m x_i \leq \omega$. A *personalized price system* is a vector $p = (p_1, \ldots, p_m)$, in which p_i is a price system for all $i \in I$. Every personalized price system p induces a *value function* $\psi_p : \sum_{i=1}^m L_1^+(\mathscr{F}_i) \to \mathbb{R}_+$, which assigns to an element x of the domain the value

$$\psi_p \cdot x = \sup \left\{ \sum_{i=1}^m p_i \cdot y_i : y \in \Pi_{i=1}^m L_1^+(\mathscr{F}_i), \sum_{i=1}^m y_i \leq x \right\}.$$

An allocation with free disposal x is said to be a *personalized equilibrium* if there exists a personalized price system p such that

$$y \in P_i(x_i) \implies \psi_p \cdot y > \psi_p \cdot x_i \tag{1}$$

for all $i \in I$, and

$$\psi_p \cdot \sum_{i=1}^{m} \alpha_i \omega_i \leq \sum_{i=1}^{m} \alpha_i \psi_p \cdot x_i \tag{2}$$

for all $\alpha \in \mathbb{R}_+^m$. Finally, it is convenient for us to define

$$\kappa_c = \operatorname{ess\,inf} \omega_c \text{ and } \kappa_{ic} = \operatorname{ess\,inf} \omega_{ic}$$

for every $i \in I$ and $c \in C$.

To ensure the existence of personalized equilibria, we need Assumption (E) stated below. It is an adaptation of assumption (A5) in *Aliprantis et al.* (2001) to our setting. We will utilize some of their results, which we present in the theorem following the statement of the assumption below. The assumption simply requires that the initial endowment of each agent is bounded away from zero in some commodity.

(E) *Boundedness:* for every $i \in I$, there exist a $c \in C$ such that $\kappa_{ic} > 0$.

Theorem 2 (*Aliprantis, Tourky, and Yannelis*, 2001)**.** *If Assumptions (A) and (E) hold, then there exist a personalized equilibrium x and a personalized price system p such that, in addition to (1) and (2), the following two properties hold:*

$$y \in P_i(x_i) \implies p_i \cdot y > p_i \cdot x_i = \psi_p \cdot x_i, \tag{3}$$

for all $i \in I$, and

$$\psi_p \cdot \omega = \sum_{i=1}^{m} p_i \cdot x_i. \tag{4}$$

We show that a personalized equilibrium x given by this theorem under independence is actually a Radner equilibrium once we limit the substitutability of one state-independent commodity by others. This is ensured by Assumption (F). We also require joint independence of agents' information, as stated in (B$'$).

(F) *Limited substitutability:* for every $i \in I$ and $c \in C$, if $x \in L_1^+(\mathscr{F}_i)$ is defined by letting $x_c(s) = \omega_{ic}(s) - \kappa_{ic}$ and, otherwise, $x_d(s) = \omega_{id}(s) + \lambda_d$ for some $\lambda_d \in [-\kappa_{id}, \kappa_d]$, then $\omega_i \in P_i(x)$.

(B$'$) *Independence:* $\mu\left(\bigcap_{i=1}^m F_i\right) = \Pi_{i=1}^m \mu(F_i)$ for all $(F_1, \ldots, F_m) \in \Pi_{i=1}^m \mathscr{F}_i$.

Theorem 3. *If Assumptions (A), (B$'$), and (F) hold, then there exists a Radner equilibrium.*

Let us present a simple example of Assumption (F) being satisfied. Consider a utility function $U_i : L_1^+(\mathscr{F}_i) \to \mathbb{R}$ defined by letting

$$U_i(x) = -\sum_{d \in C} \int_S e^{-\rho x_d} d\mu$$

for some scalar $\rho > 0$. Suppose that P_i derives from this utility function, i.e. $y \in P_i(x)$ if and only if $U_i(y) > U_i(x)$. Also, suppose that there exists an $F \in \mathscr{F}_i$ such that $\mu(F) > 0$ and $\omega_{id}(s) = \kappa_{id}$ for almost all $s \in S$ and for all $d \in C$. Since U_i is monotone, Assumption (F) holds if and only if it is satisfied when we let $\lambda_d = \kappa_d$ for all d. Consider the corresponding $x \in L_1^+(\mathscr{F}_i)$ for any $c \in C$. Observe that

$$\begin{aligned} U_i(x) &= -\left(\int_F e^{-\rho x_c} d\mu + \int_{S \setminus F} e^{-\rho x_c} d\mu + \sum_{d \neq c} \int_S e^{-\rho x_d} d\mu\right) \\ &\leq -\int_F e^{-\rho x_c} d\mu = -\mu(F). \end{aligned}$$

Also, notice that $U_i(\omega_i) \geq -\sum_{d \in C} e^{-\rho \kappa_{id}}$. Our last two observations show that we have $U_i(\omega_i) > U_i(x)$ if

$$\sum_{d \in C} e^{-\rho \kappa_{id}} < \mu(F).$$

This inequality is satisfied for all sufficiently large ρ provided that $\kappa_{id} > 0$ for all d. In other words, Assumption (F) holds for this particular agent i if $\kappa_{id} > 0$ for all d and ρ is sufficiently large, regardless of other agents' characteristics.

4 Common Knowledge of Total Endowment

In this section we prove that Radner equilibrium exists when the total endowment ω is common knowledge. Formally expressed in (G) below, this assumption holds if and only if ω belongs to the informationally constrained consumption set $L_1(\mathscr{F}_i)$ of every agent i.

(G) *Common knowledge:* ω_c is $\mathscr{F}_i$-measurable for all $i \in I$ and $c \in C$.

To get existence, we also need Assumptions (H1) – (H6), where we let $E(P_i(x)|\mathscr{G}) = \{E(y|\mathscr{G}) : y \in P_i(x)\}$, for all $x \in L_1^+(\mathscr{F}_i)$. These assumptions are somewhat unusual, but Proposition 1 shows that they are implied by standard assumptions listed in (A) provided that the risk aversion condition stated in (H7) holds. The term 'risk averse' is justified, because the conditional expectation operator preserves the mean and never increases the variance of a random variable (*Abramovich and Aliprantis*, 2002, Lemma 5.38).

(H) The following is true for every $i \in I$, some sub-σ-algebra $\mathscr{G}$ of $\bigcap_{j=1}^m \mathscr{F}_j$ such that ω_c is $\mathscr{G}$-measurable for all $c \in C$, and some $v \in (L_1(\mathscr{G}))^m$ satisfying $\sum_{j=1}^m v_j \leq \omega$ and $v_j > 0$ for all $j \in I$.

(1) P_i is *conditionally irreflexive*, i.e. $x \notin E(P_i(x)|\mathscr{G})$ for all $x \in L_1^+(\mathscr{G})$.

(2) P_i is *conditionally convex-valued*, i.e. $E(P_i(x)|\mathscr{G})$ is a convex set for all $x \in L_1^+(\mathscr{G})$.

(3) P_i is *conditionally strictly monotone*, i.e. $x \in L_1^+(\mathscr{G})$ implies $x + y \in E(P_i(x)|\mathscr{G})$ for all $y \in L_1^+(\mathscr{G}) \setminus \{0\}$.

(4) P_i has *open conditional values*, i.e. $E(P_i(x)|\mathscr{G})$ is open in $L_1^+(\mathscr{G})$, relative to a linear topology on $L_1(\mathscr{G})$, for all $x \in L_1^+(\mathscr{G})$.

(5) P_i has *weakly open conditional lower sections*, i.e. for every $x \in L_1^+(\mathscr{G})$ the set $\{y \in L_1^+(\mathscr{G}) : x \in E(P_i(y)|\mathscr{G})\}$ is weakly open in $L_1^+(\mathscr{G})$.

(6) P_i is *conditionally proper* in the sense that there exists a convex-valued correspondence $\tilde{P}_i : L_1^+(\mathscr{G}) \twoheadrightarrow L_1(\mathscr{G})$ such that for each $x \in L_1^+(\mathscr{G})$

  (i) $x + v_i$ is an interior point of $\tilde{P}_i(x)$ and

  (ii) $\tilde{P}_i(x) \cap L_1^+(\mathscr{G}) = E(P_i(x)|\mathscr{G})$.

(7) P_i is *risk averse* in the sense that $E(P_i(x)|\mathscr{G}) \subset P_i(x)$ for all $x \in L_1^+(\mathscr{G})$.

Proposition 1. *If Assumption (G) is satisfied, then the following statements are true:*

(1) (A1) and (H7) together imply (H1);

(2) (A2) implies (H2);

(3) (A3) implies (H3);

(4) (A4) and (H7) together imply (H4);

(5) (A5) implies (H5);

(6) (A6) with $v \in (L_1(\mathscr{G}))^m$ and (H7) together imply (H6).

Our approach in the following theorem is to find an equilibrium in the projection of our economy into a smaller commodity space, $L_1(\mathscr{G})$. The risk aversion assumption ensures that this projected economy is well-behaved, in the sense of meeting Assumptions (H1) – (H6), and has an equilibrium. We then show that this equilibrium is also a Radner equilibrium of the original economy.

Theorem 4. *If Assumptions (G), (H1) – (H6) hold and $\omega_i > 0$ for all $i \in I$, then there exists a Radner equilibrium.*

Examples

In this subsection we suppose that there is a single commodity per state, i.e. the set C is a singleton. We give examples of preferences that satisfy and violate our assumptions.

Assumption (H7) is satisfied for any sub-σ-algebra $\mathscr{G}$ of $\mathscr{F}_i$ if P_i has an expected utility representation with a concave Bernoulli function $u_i : \mathbb{R} \to \mathbb{R}$, i.e. $y \in P_i(x)$ if and only if $E(u_i \circ y) > E(u_i \circ x)$. Indeed, the Jensen's inequality yields $u_i \circ E(y|\mathscr{G}) \geq E(u_i \circ y|\mathscr{G})$, and consequently

$$E(u_i \circ E(y|\mathscr{G})) \geq E(E(u_i \circ y|\mathscr{G})) = E(u_i \circ y) > E(u_i \circ x). \tag{5}$$

The case of risk averse expected utility is important because even in such a simple setup equilibrium may fail to exist without the common knowledge assumption (*Tourky and Yannelis*, 2003; *Podczeck et al.*, 2008).

More generally, in the next paragraph we will show that Assumption (H7) holds if P_i can be represented by an implicitly separable utility function (*Epstein*, 1986; *Dekel*, 1986). Such a utility function $U_i : L_1^+(\mathscr{F}_i) \to \mathbb{R}$ is defined implicitly by

$$U_i(x) = E(v_i(\cdot, U_i(x)) \circ x) \tag{6}$$

with some $v_i : \mathbb{R}^2 \to \mathbb{R}$ such that $v_i(\cdot, \beta)$ is concave and strictly increasing for all $\beta \in U_i\left(L_1^+(\mathscr{F}_i)\right)$ and such that $v_i(\alpha, \cdot)$ is decreasing for all $\alpha \in \mathbb{R}_+$. For instance, let $v_i(\alpha, \beta) = -e^{\alpha\beta}$. For each $x \in L_1^+(\mathscr{F}_i)$, the expected value $E(v_i(\cdot, \beta) \circ x)$ is continuous in β on $[-1, 0]$ by the Lebesgue dominated convergence theorem. Thus the intermediate value theorem gives us a $U_i(x) \in \mathbb{R}$ solving equation (6), and it is readily seen that this solution is unique. *This utility function cannot be always represented by expected utility maximization*, i.e. it may be impossible to find a Bernoulli function $u_i : \mathbb{R}_+ \to \mathbb{R}$ such that $U_i(x) > U_i(y)$ if and only if $E(u_i \circ x) > E(u_i \circ y)$. To see this, let $S = \{1, 2, 3\}$, $\mathscr{F}_i = 2^S$, and $\mu(\{1\}) = \mu(\{2\}) = \mu(\{3\}) = \frac{1}{3}$. In this case $U_i(x)$ solves

$$e^{x_1 U_i(x)} + e^{x_2 U_i(x)} + e^{x_3 U_i(x)} + 3U_i(x) = 0. \tag{7}$$

Differentiating the left-hand side of this equation with respect to $U_i(x)$ yields

$$x_1 e^{x_1 U_i(x)} + x_2 e^{x_2 U_i(x)} + x_3 e^{x_3 U_i(x)} + 3 > 0.$$

Thus the implicit function theorem allows us computing

$$\frac{\partial U_i(x)}{\partial x_3} = -\frac{U_i(x)\, e^{x_3 U_i(x)}}{x_1 e^{x_1 U_i(x)} + x_2 e^{x_2 U_i(x)} + x_3 e^{x_3 U_i(x)} + 3}$$

for all $x \in \mathbb{R}^3_+$. Let $y = (1, 1, 1) \in \mathbb{R}^3_+$ and notice that $U_i(y) = -e^{U_i(y)}$. It is easy to check that $-\frac{3}{5} < U_i(y) < -\frac{14}{25} < -\frac{5}{9}$. It follows that $e^{3U_i(y)} > e^{-\frac{9}{5}}$ and that $1 - 2e^{U_i(y)} + e^{-\frac{9}{5}} > 1 - 2e^{-\frac{5}{9}} + e^{-\frac{9}{5}} > 0$. Combining these inequalities yields $1 - 2e^{U_i(y)} + e^{3U_i(y)} > 0$, which implies

$$1 + e^{3U_i(y)} + e^{U_i(y)} + 3U_i(y) > e^{U_i(y)} + e^{U_i(y)} + e^{U_i(y)} + 3U_i(y) = 0.$$

Letting $z_0 = (0, 3, 1) \in \mathbb{R}^3_+$, the previous inequality and equation (7) imply $U_i(z_0) < U_i(y)$. Since for $z_1 = (1, 3, 1) \in \mathbb{R}^3_+$ we have $U_i(z_1) > U_i(y)$, the intermediate value theorem gives us a $\gamma \in (0, 1)$ such that $z_\gamma = (\gamma, 3, 1) \in \mathbb{R}^3_+$ is indifferent to y, i.e. $U_i(z_\gamma) = U_i(y)$. We will show that in this case

$$\frac{\partial U_i(y)}{\partial x_3} = \frac{-U_i(y)\, e^{U_i(y)}}{2e^{U_i(y)} + e^{U_i(y)} + 3} \neq \frac{-U_i(y)\, e^{U_i(y)}}{\gamma e^{\gamma U_i(y)} + 3e^{3U_i(y)} + e^{U_i(y)} + 3} = \frac{\partial U_i(z_\gamma)}{\partial x_3},$$

which is inconsistent with the existence of an additively separable representation. Simply notice that $e^{U_i(y)} = -U_i(y) > \frac{14}{25} > 3e^{-\frac{42}{25}} > 3e^{3U_i(y)}$ and that $e^{U_i(y)} > \gamma e^{\gamma U_i(y)}$, because $\gamma e^{\gamma U_i(y)}$ is strictly increasing in γ on $[0,1]$.

If P_i can be represented by an implicitly separable utility function U_i as in (6), then $y \in P_i(x)$ if and only if

$$E(v_i(\cdot, U_i(x)) \circ y) > E(v_i(\cdot, U_i(x)) \circ x), \tag{8}$$

for all $x, y \in L_1^+(\mathscr{F}_i)$. Thus Assumption (H7) is implied by the concavity of $v_i(\cdot, U_i(x))$, as in (5), for any sub-σ-algebra $\mathscr{G}$ of $\mathscr{F}_i$. Assumptions listed in (A) are also satisfied. (A1) holds by the existence of utility representation. (A2) is also implied by the concavity of $v_i(\cdot, U_i(x))$. (A3) is satisfied because $v_i(\cdot, U_i(x))$ is strictly increasing. (A4) holds because the left-hand side of (8) is continuous in y on $L_1^+(\mathscr{F}_i)$ (*Balder and Yannelis*, 1993, Corollary 2.11). (A5) is satisfied because the left-hand side of (8) is weakly upper semicontinuous in y on $L_1^+(\mathscr{F}_i)$ (*Balder and Yannelis*, 1993, Theorem 2.8). Finally, let us show that (A6) also holds for any $v_i \in L_1^+(\mathscr{F}_i)$ such that $v_i > 0$. Let $\beta \in \mathbb{R}_{++}$ be a supergradient of $v_i(\cdot, U_i(x))$ at zero. Now define a concave function $u : \mathbb{R} \to \mathbb{R}$ by letting $u(\alpha) = v_i(\alpha, U_i(x))$ if $\alpha \geq 0$ and $u(\alpha) = \beta\alpha + v_i(0, U_i(x))$ otherwise. A suitable correspondence $\hat{P}_i$ is obtained by letting $\hat{P}_i(x) = \{y \in L_1(\mathscr{F}) : E(u \circ y) > E(u \circ x)\}$.

We must admit that (H7) is a strong assumption, because it may fail when agents are subjective expected utility maximizers with priors $p_i \in L_\infty(S, \mathscr{F}, \mu)$, e.g. $y \in P_i(x)$ if and only if $E(p_i y) > E(p_i x)$. In this case even conditional irreflexivity (H1), which is implied by (H7) according to Proposition 1, may fail. To see this, suppose that $\mathscr{G} = \{\emptyset, S\}$ and we can pick an essentially bounded $p_i \in L_1^+(\mathscr{F}_i)$ having nonzero variance. Since $E(p_i^2) - E(p_i E(p_i|\mathscr{G})) = E(p_i^2) - (E(p_i))^2 > 0$, we can find an $x \in L_1^+(\mathscr{F}_i)$ such that $E(p_i^2) > E(p_i x) > E(p_i E(p_i|\mathscr{G}))$, and thus $x \in E(P_i(x)|\mathscr{G})$.

5 Proofs

Proof of Theorem 3

First we show that

$$\operatorname{ess\,inf} y + \operatorname{ess\,inf} z = \operatorname{ess\,inf}(y+z), \tag{9}$$

whenever $y, z \in L_1(S, \mathscr{F}, \mu)$ are positive and independent (generating independent σ-algebras). Clearly, we have $\operatorname{ess\,inf} y + \operatorname{ess\,inf} z \leq \operatorname{ess\,inf}(y+z)$. We prove that $\operatorname{ess\,inf} y + \operatorname{ess\,inf} z \geq \operatorname{ess\,inf}(y+z)$ by demonstrating the following: for every scalar $\varepsilon > 0$, there exists an $F \in \mathscr{F}$ such that $\mu(F) > 0$ and $y(s) + z(s) \leq \operatorname{ess\,inf} y + \operatorname{ess\,inf} z + \varepsilon$ for almost all $s \in F$. We can find a set Y, belonging to the sub-σ-algebra $\mathscr{Y}$ of $\mathscr{F}$ generated by y, such that $\mu(Y) > 0$ and $y(s) \leq \operatorname{ess\,inf} y + \frac{\varepsilon}{2}$ for almost all $s \in Y$. Also, we can find a set Z, belonging to the sub-σ-algebra $\mathscr{Z}$ of $\mathscr{F}$ generated by z, such that $\mu(Z) > 0$

and $z(s) \leq \operatorname{ess\,inf} z + \frac{\varepsilon}{2}$ for almost all $s \in Z$. The independence implies that $\mu(Z \cap Y) = \mu(Z)\mu(Y) > 0$. Let $F = Z \cap Y$.

We also show that

$$\operatorname{ess\,inf} y \geq \operatorname{ess\,sup} z, \tag{10}$$

whenever $y, z \in L_1(S, \mathscr{F}, \mu)$ are independent and satisfy $y \geq z \geq 0$. Suppose, by way of contradiction, that $\operatorname{ess\,inf} y < \operatorname{ess\,sup} z \leq +\infty$. Pick an $\alpha \in (\operatorname{ess\,inf} y, \operatorname{ess\,sup} z)$. We can find a set Y, belonging to the sub-σ-algebra $\mathscr{Y}$ of $\mathscr{F}$ generated by y, such that $\mu(Y) > 0$ and $y(s) < \alpha$ for almost all $s \in Y$. Also, we can find a set Z, belonging to the sub-σ-algebra $\mathscr{Z}$ of $\mathscr{F}$ generated by z, such that $\mu(Z) > 0$ and $z(s) > \alpha$ for almost all $s \in Z$. Notice that $z(s) > \alpha > y(s)$ for almost all $s \in Z \cap Y$. Since $y \geq z$, it must be the case that $\mu(Z \cap Y) = 0 < \mu(Z)\mu(Y)$. This contradicts the independence of y and z.

Now we let L^+ denote the positive cone of $L = \sum_{i=1}^m L_1(\mathscr{F}_i)$ and argue that

$$L^+ \subset \sum_{i=1}^m L_1^+(\mathscr{F}_i). \tag{11}$$

Define $M = \{y \in \Pi_{i=1}^m L_1(\mathscr{F}_i) : \sum_{i=1}^m y_i \geq 0\}$, and consider any $y \in M$. We have

$$\sum_{i=1}^m y_i^+ \geq \sum_{i=1}^m y_i^-. \tag{12}$$

Pick any $j \in I$. For every $c \in C$, we can find an $F_c \in \mathscr{F}_j$ such that $y_{jc}^-(s) > 0 = y_{jc}^+(s)$ for almost all $s \in F_c$ and $y_{jc}^-(s) = 0$ for almost all $s \in S \backslash F_c$. In view of (12), this observation shows that

$$\sum_{i \neq j} y_i^+ \geq y_j^-. \tag{13}$$

Define $y_{-jc}^+ = \sum_{i \neq j} y_{ic}^+$, for $c \in C$, and let $\mathscr{F}_J$ denote the smallest σ-algebra containing $\bigcup_{i \in J} \mathscr{F}_i$, for $J \subset I$. For all $k \in I$ and $J \subset I$, Assumption (B′) implies that $\mathscr{F}_k$ and $\mathscr{F}_{J \backslash k}$ are independent (*Skorokhod*, 2004, Corollary 3.1.1). It follows that $\sum_{i \in J \backslash \{k\}} y_{ic}^+$ and y_{kc}^+ (or y_{kc}^-) are independent. Combining this observation with (9), (10), and (13), we see that

$$\sum_{i \neq j} \operatorname{ess\,inf} y_{ic}^+ = \operatorname{ess\,inf} y_{-jc}^+ \geq \operatorname{ess\,sup} y_{jc}^-,$$

for all $c \in C$. Define a $z \in \Pi_{i=1}^m L_1(\mathscr{F}_i)$ by letting $z_{ic}(s) = y_{ic}(s) - \operatorname{ess\,inf} y_{ic}^+$ for $i \neq j$, and $z_{jc}(s) = y_{jc}(s) + \operatorname{ess\,inf} y_{-jc}^+$, for all $c \in C$ and $s \in S$. Letting $T_j y = z$ defines a transformation $T_j : M \to M$. This transformation satisfies

(i) $\sum_{i=1}^m z_i = \sum_{i=1}^m y_i$,

(ii) $z_j \geq 0$, and

(iii) $y_i \geq 0 \implies z_i \geq 0$, for all $i \in I$,

for all $y \in M$ and $z = T_j y$. Consider any $y \in M$ and notice that $z = T_1 T_2 \dots T_m y \in \Pi_{i=1}^m L_1^+ (\mathscr{F}_i)$ is such that $\sum_{i=1}^m z_i = \sum_{i=1}^m y_i$. This proves our claim.

Next we define $\mathscr{K} = \{\emptyset, S\}$ and argue that each $y \in L$ has a unique decomposition

$$y = E\left(y|\mathscr{K}\right) + \sum_{i=1}^{m} z_i, \tag{14}$$

such that $z_i \in L_1 (\mathscr{F}_i)$ and $E(z_i) = 0$ for each i. A quick thought confirms that at least one such decomposition exists. So let us consider any two decompositions

$$y = E\left(y|\mathscr{K}\right) + \sum_{i=1}^{m} z_i' = E\left(y|\mathscr{K}\right) + \sum_{i=1}^{m} z_i'' \tag{15}$$

with $z_i', z_i'' \in L_1 (\mathscr{F}_i)$ and $E(z_i') = E(z_i'') = 0$ for each i. Taking conditional expectations in (15) with respect to any $\mathscr{F}_j$ shows that $z_j' = z_j''$, because $E(z_{ic}|\mathscr{F}_j)$ is equal to $E(z_{ic}) = 0$ almost everywhere for all $i \neq j$ and for all $c \in C$ due to Assumption (B′). This proves our claim.

Now notice that Assumption (E) holds, for otherwise Assumption (F) would yield $\omega_i \in P_i(\omega_i)$, which contradicts (A1). This means that we can use Theorem 2 to obtain a personalized equilibrium x and a personalized price system p satisfying (2), (3), and (4).

We proceed to demonstrate that x is an allocation, i.e. we have

$$\sum_{i=1}^{m} x_i = \omega. \tag{16}$$

Suppose, by way of contradiction, that x is not an allocation. Since x is an allocation with free disposal, we have $0 < \omega - \sum_{i=1}^m x_i \in L^+$. Now (11) yields a $y \in \Pi_{i=1}^m L_1^+ (\mathscr{F}_i)$ such that

$$\sum_{i=1}^{m} y_i = \omega - \sum_{i=1}^{m} x_i > 0. \tag{17}$$

It must be the case that $y_j > 0$ for some j. Using (3) and Assumption (A3), we see that $p_j \cdot y_j > 0$. Since $y_j + \sum_{i=1}^m x_i \leq \omega$, it follows that

$$\psi_p \cdot \omega \geq p_j \cdot y_j + \sum_{i=1}^{m} p_i \cdot x_i > \sum_{i=1}^{m} p_i \cdot x_i,$$

which contradicts (4).

Our next step is to show that

$$\operatorname{ess\,inf} x_{ic} > 0 \tag{18}$$

for all $i \in I$ and $c \in C$. Suppose, by way of contradiction, that $\operatorname{ess\,inf} x_{ic} = 0$ for some i and c. Assumption (B′) implies that $E\left(x_{jd}|\mathscr{F}_i\right)$ is equal to $E\left(x_{jd}\right)$ almost everywhere for all $j \neq i$ and $d \in C$. Now taking conditional expectations with respect to $\mathscr{F}_i$ on both sides of $\sum_{j=1}^{m} x_{jd} = \sum_{j=1}^{m} \omega_{jd}$ shows that

$$x_{id}(s) + \sum_{j \neq i} E\left(x_{jd}\right) = \omega_{id}(s) + \sum_{j \neq i} E\left(\omega_{jd}\right) \tag{19}$$

for almost all $s \in S$. Consequently, we have $x_{id}(s) = \omega_{id}(s) + \lambda_d$ for some $\lambda_d \in \mathbb{R}$. Now $\lambda_d \leq x_{id}(s) \leq \omega(s)$ reveals that $\lambda_d \leq \kappa_d$. On the other hand, we have $0 \leq \operatorname{ess\,inf} x_{id} = \operatorname{ess\,inf} \omega_{id} + \lambda_d$, which implies that $\lambda_d \geq -\kappa_{id}$. Since $\operatorname{ess\,inf} x_{ic} = 0$, we see that actually $\lambda_c = -\kappa_{ic}$. Our last three observations were ment to verify that x_i lies within the orbit of Assumption (F), which implies that $\omega_i \in P_i(x_i)$. This means that x is not individually rational. However, it must be individually rational by Lemmas 6.2 and 6.4 of *Aliprantis et al.* (2001). This contradiction proves our claim.

Letting K be the subspace of all $y \in L_1(\mathscr{F})$ such that y_c is constant almost everywhere for all $c \in C$, we argue that

$$p_i \cdot y = p_j \cdot y \tag{20}$$

for all $y \in K$ and $i, j \in I$. Suppose, by way of contradiction, that $p_i \cdot y > p_j \cdot y$ for some $y \in K$ and $i, j \in I$. It must be the case that $E\left(p_{ic} y_c\right) > E\left(p_{jc} y_c\right)$ for some $c \in C$. Clearly, we have $y_c \neq 0$. We may assume that $y_c > 0$. Due to (18), we can pick a scalar $\alpha > 0$ such that $\alpha y_c < x_{jc}$. Now define an allocation z by letting $z_{ic} = x_{ic} + \alpha y_c$, $z_{jc} = x_{jc} - \alpha y_c$, and $z_{kd} = x_{kd}$ for $k \notin \{i, j\}$ and $d \neq c$. The fact that

$$\psi_p \cdot \omega \geq \sum_{k=1}^{m} p_k \cdot z_k > \sum_{k=1}^{m} p_k \cdot x_k$$

contradicts (4).

Now we define a linear functional q' on L by letting

$$q' \cdot y = p_1 \cdot E\left(y|\mathscr{K}\right) + \sum_{i=1}^{m} p_i \cdot z_i,$$

where z_i are uniquely chosen as in (14). Using (20), we see that

$$q' \cdot y = p_i \cdot E\left(y|\mathscr{K}\right) + p_i \cdot \left(y - E\left(y|\mathscr{K}\right)\right) = p_i \cdot y \tag{21}$$

for all $i \in I$ and $y \in L_1(\mathscr{F}_i)$.

Next we show that q' is (weakly) continuous. Consider a net y^λ in L converging weakly to a point $y \in L$. Consider also the respective decompositions

$y^\lambda = E\left(y^\lambda \middle| \mathscr{K}\right) + \sum_{i=1}^m z_i^\lambda$ such that $z_i^\lambda \in L_1(\mathscr{F}_i)$ and $E\left(z_i^\lambda\right) = 0$ for each i. Notice that $E\left(y_c^\lambda \middle| \mathscr{K}\right)$ is equal to $E\left(y_c^\lambda\right)$ almost everywhere, for all c and λ, and that $E\left(y_c^\lambda\right)$ converges to $E(y_c)$. These observations imply that $E\left(y^\lambda \middle| \mathscr{K}\right)$ converges weakly to $E(y|\mathscr{K})$. Consequently, the net $z^\lambda = \sum_{i=1}^m z_i^\lambda$ also converges weakly to some $z \in L$. Pick any $i \in I$ and a price system q such that q_c is $\mathscr{F}_i$-measurable for all $c \in C$. For every $j \neq i$, we have

$$E\left(q_c z_{jc}^\lambda\right) = E\left(E\left(q_c z_{jc}^\lambda \middle| \mathscr{F}_i\right)\right) = E\left(q_c E\left(z_{jc}^\lambda \middle| \mathscr{F}_i\right)\right) = 0,$$

since $E\left(z_{jc}^\lambda \middle| \mathscr{F}_i\right)$ is equal to $E\left(z_{jc}^\lambda\right) = 0$ almost everywhere due to Assumption (B′). This implies that

$$q \cdot z^\lambda = \sum_{j=1}^m q \cdot z_j^\lambda = q \cdot z_i^\lambda,$$

and it follows that z_i^λ converges weakly to $E(z|\mathscr{F}_i)$. Now we see that $y = E(y|\mathscr{K}) + \sum_{i=1}^m E(z|\mathscr{F}_i)$, and $q' \cdot y^\lambda = p_1 \cdot E\left(y^\lambda \middle| \mathscr{K}\right) + \sum_{i=1}^m p_i \cdot z_i^\lambda$ converges to $q' \cdot y = p_1 \cdot E(y|\mathscr{K}) + \sum_{i=1}^m p_i \cdot E(z|\mathscr{F}_i)$. This proves that q' is indeed continuous.

We obtain a price system q by taking any continuous extension of q' to all of $L_1(\mathscr{F})$. Using (3) and (21), we see that

$$y \in P_i(x_i) \implies q \cdot y > q \cdot x_i, \tag{22}$$

for all $i \in I$.

We complete the proof by showing that

$$q \cdot x_i \leq q \cdot \omega_i,$$

for all $i \in I$. Suppose, by way of contradiction, that $q \cdot x_i > q \cdot \omega_i$ for some i. Using (21), (3), and (2), we also see that

$$q \cdot x_j = p_j \cdot x_j = \psi_p \cdot x_j \geq \psi_p \cdot \omega_j \geq p_j \cdot \omega_j = q \cdot \omega_j,$$

for all $j \in I$. It follows that $\sum_{j=1}^m q \cdot x_j > \sum_{j=1}^m q \cdot \omega_j$, which contradicts (16).

Proof of Theorem 1

Let p_i, for each i, be price systems given by Assumption (C). Consider the proof of Theorem 3 with $x = (\omega_1, \ldots, \omega_m)$ and $p = (p_1, \ldots, p_m)$. Notice that (20) holds because $E(p_i) = E(p_j)$ for all $i, j \in I$. This means that we can advance to obtain a price system q satisfying (22). Since $x_i = \omega_i$ for all i, we conclude that x is a Radner equilibrium.

To demonstrate uniqueness, consider an arbitrary Radner equilibrium x supported by a price system p. As in (19), we see that

$$x_i(s) + \sum_{j \neq i} E(x_j) = \omega_i(s) + \sum_{j \neq i} E(\omega_j)$$

for almost all $s \in S$ for all $i \in I$. Consequently, we have $x_i(s) = \omega_i(s) + E(x_i) - E(\omega_i)$. It must be the case that $E(x_i) - E(\omega_i) \geq 0$, for otherwise Assumption (D) implies that $\omega_i \in P_i(x_i)$, and hence $p \cdot \omega_i > p \cdot \omega_i$. But if $E(x_i) - E(\omega_i) > 0$ for some i, then $E\left(\sum_{j=1}^m x_j\right) > E\left(\sum_{j=1}^m \omega_j\right)$, which is impossible. We conclude that $E(x_i) = E(\omega_i)$, and it follows that $x_i = \omega_i$ for all i.

Proof of Proposition 1

By Assumption (G), a suitable σ-algebra $\mathscr{G}$ exists, e.g. $\mathscr{G} = \bigcap_{j=1}^m \mathscr{F}_j$.

(4) Simply observe that $E(P_i(x)|\mathscr{G}) = P_i(x) \cap L_1^+(\mathscr{G})$.

(5) Similarly, observe that

$$\left\{y \in L_1^+(\mathscr{G}) : x \in E(P_i(y)|\mathscr{G})\right\} = \bigcup_{z \in E(\cdot|\mathscr{G})^{-1}(x)} \left\{y \in L_1^+(\mathscr{G}) : z \in P_i(y)\right\}$$
$$= \left(\bigcup_{z \in E(\cdot|\mathscr{G})^{-1}(x)} P_i^{-1}(z)\right) \cap L_1^+(\mathscr{G}).$$

(6) Let $\tilde{P}_i(x) = \hat{P}_i(x) \cap L_1(\mathscr{G})$. Clearly, $\tilde{P}_i$ is convex-valued, and the point $x + v_i$ belongs to the interior of $\tilde{P}_i(x)$. Also, we have

$$\tilde{P}_i(x) \cap L_1^+(\mathscr{G}) = \hat{P}_i(x) \cap L_1^+(\mathscr{G}) = P_i(x) \cap L_1^+(\mathscr{G}) = E(P_i(x)|\mathscr{G}).$$

Proof of Theorem 4

We will first find an equilibrium in our economy projected into a smaller commodity space, $L_1(\mathscr{G})$. In this economy agents' consumption sets are identical and coincide with the positive cone $X = L_1^+(\mathscr{G})$ of the commodity space. Agent i's preference correspondence is $Q_i : X \twoheadrightarrow X$ defined by $Q_i(x) = E(P_i(x)|\mathscr{G})$, and the agent's initial endowment is $E(\omega_i|\mathscr{G})$. What we have just constructed is a classical Walrasian economy (see *Aliprantis et al.*, 2001, Section 9.1). Using Assumptions (H1) – (H6), utilizing the fact that $\omega_i > 0$ for all i, and invoking Corollary 9.2 of *Aliprantis et al.* (2001), we see that this economy has a Walrasian equilibrium $x \in X^m$. It is supported by a price system p such that p_c is $\mathscr{G}$-measurable for all $c \in C$. Consequently, we have

(i) $\sum_{i=1}^m x_i = \sum_{i=1}^m E(\omega_i|\mathscr{G}) = \omega$,

(ii) $p \cdot x_i = p \cdot E(\omega_i|\mathscr{G}) = p \cdot \omega_i$, for all i, and

(iii) $y \in Q_i(x_i)$ implies $p \cdot y > p \cdot E(\omega_i|\mathscr{G}) = p \cdot \omega_i$, for all i.

The allocation x is in fact a Radner equilibrium, because $y \in P_i(x_i)$ implies $E(y|\mathscr{G}) \in Q_i(x_i)$, and $p \cdot y = p \cdot E(y|\mathscr{G}) > p \cdot \omega_i$.

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