Q 1. Consider the game given by the tree below and find all Nash equilibria.



Q 2. Find all subgame perfect equilibria of the game in question 1.

Q 3. Let V and W be the linear polyhedral cones generated by the rows and the columns respectively of an $n \times m$ matrix A with linearly independent columns, i.e.

$$V = \left\{ Ay : y \in \mathbb{R}_+^m \right\} \text{ and } W = \left\{ A^t x : x \in \mathbb{R}_+^n \right\}.$$

Use results from game theory to show that

$$V \cap \mathbb{R}^n_- = \{0\} \implies W \cap \mathbb{R}^m_+ \neq \{0\}.$$

Q 4. Consider the game in strategic form given by the table below and find all mixed strategy Nash equilibria.

Player 2

$$A \quad B$$

Player 1 $A \quad 6,1 \quad 0,0$
 $B \quad 0,0 \quad 1,6$

Q 5. Let Γ be an *N*-person game in strategic form. Prove that in a mixed equilibrium each player is indifferent between the actions she plays with positive probability (in equilibrium).

Q 6. Using Brower's fix point theorem, prove that all 2-person zero sum games in strategic form have a Nash equilibrium.

Q 7. Let X be a topological space and F a correspondence from X into the power set of X. Define the notion of upper-semicontinuity. Give and state an alternative characterisation of upper-semicontinuity in case X is a metric space.

Q 8. Let X be a nonempty subset of \mathbb{R}^n . Let $F: X \to \mathcal{P}(X)$ be a correspondence with nonempty, compact, and convex values. Show that $x^* \in X$ is a fixed point of F if x^* is an element of the closure of any convex set $V \subset \mathbb{R}^n$ such that the interior of V includes $F(x^*)$.

Q 9. State and prove the fixed point property of Nash equilibrium.